

# Emerging Market Business Cycles with Heterogeneous Agents<sup>\* †</sup>

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## Abstract

A central question in open economy macroeconomics is how to explain excess consumption volatility observed in emerging market business cycles. Existing explanations crucially depend on households' permanent income hypothesis (PIH) behavior, while the high marginal propensity to consume (MPC) observed in emerging market micro data suggests a strong deviation from the PIH. This paper explains emerging market business cycles by estimating a two-asset heterogeneous-agent small open economy model where MPC is as high as the micro estimates due to financial friction in the form of asset illiquidity. A conventional mechanism through which a representative-agent model explains the business cycles does not work in the heterogeneous-agent model because of financial friction and precautionary saving. Instead, a new mechanism explains the business cycles in which high MPC and correspondingly strong financial friction and precautionary saving play essential roles. When MPC is lowered to the U.S. level via recalibration, excess consumption volatility disappears.

**JEL classification:** E21, E32, F41, D31

**Keywords:** emerging economy, business cycle, heterogeneous agents, MPC, financial friction

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# 1 Introduction

A central question in open economy macroeconomics is how to explain ‘excess consumption volatility’ of emerging economies: consumption is more volatile than output in emerging economies, while it is not in developed economies. Extensive literature is devoted to explaining excess consumption volatility, and the dominant modeling framework is representative-agent small open economy (RASOE) models. At the heart of these models, representative households optimize according to the permanent income hypothesis (PIH). Importantly, widely accepted mechanisms for excess consumption volatility in the literature, such as the permanent income effect of a trend shock ([Aguiar and Gopinath, 2007](#)) and the intertemporal substitution effect of interest rate fluctuations ([Neumeyer and Perri, 2005](#)), crucially depend on the household PIH behavior.

However, micro data suggest that household consumption behavior deviates significantly from the PIH in emerging economies, and the deviation is greater than that in developed economies. Under the PIH, the marginal propensity to consume (MPC) out of a transitory income shock is essentially zero. However, when [Hong \(2023\)](#) estimates MPC by applying a standard method (developed by [Blundell, Pistaferri, and Preston \(2008\)](#)) to a Peruvian household survey, he finds that Peruvian MPCs are substantially higher than U.S. MPCs, which are already greater than zero.

Motivated by this observation, this paper revisits emerging market business cycles through the lens of a heterogeneous-agent small open economy (HASOE) model in which household MPCs are as high as the empirical estimates. To this end, I incorporate [Kaplan, Moll, and Violante \(2018\)](#)’s two-asset household heterogeneity over liquid and illiquid assets into a standard RASOE model.<sup>1</sup> The financial friction in illiquid asset trading is calibrated (jointly with the time discount factor) such that both household MPC and wealth are empirically realistic. Then, I take the HASOE model to Peruvian macro data through Bayesian estimation to explain emerging market business cycles. For comparison, I also estimate the corresponding RASOE model.

I report three main findings. First, the RASOE model explains Peruvian macro data through the conventional [Aguiar and Gopinath \(2007\)](#) mechanism, while this mechanism does not work in the HASOE model. In the RASOE model, the [Aguiar and Gopinath \(2007\)](#) mechanism operates as follows: when a trend shock hits the economy, earnings mildly jump on impact but grow strongly in the future; the permanent income effect of the future earnings growth drives a strong consumption response, generating excess consumption volatility. In the HASOE model, the consumption response to a trend shock is muted for two reasons. (i) Households put most of their savings in illiquid assets, and thus it is difficult for them to borrow from the future by cashing out assets. As a result, despite a large permanent income increase, households cannot increase their consumption

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<sup>1</sup>I adopt the two-asset heterogeneity for two reasons. First, it allows me to capture both realistically high MPCs and the correct amount of aggregate wealth. Second, in reality, households in emerging economies put most of their savings in nonfinancial, illiquid assets ([Badarinza, Balasubramaniam, and Ramadorai, 2019](#)).

accordingly. (ii) The future aggregate earnings growth also means that households must face a greater idiosyncratic risk in the future and thus enhance their precautionary saving. The enhanced precautionary saving further depresses a consumption response to a trend shift.

Second, the HASOE model, once augmented with a financial friction shock<sup>2</sup>, can successfully explain emerging market business cycles through a new mechanism in which households' high MPC and correspondingly strong financial friction and precautionary saving play essential roles. Specifically, large consumption fluctuations are driven by two channels: (i) in the face of heightened financial friction, households substantially reduce their consumption because of both aggravated consumption smoothing failure and enhanced precautionary saving<sup>3</sup>; (ii) when a stationary productivity shock hits the economy, households' individual income fluctuates, and the income fluctuations are strongly translated into consumption fluctuations due to their high MPC.

Third, to evaluate the quantitative importance of the high MPC and correspondingly strong financial friction and precautionary saving, I conduct a counterfactual experiment in which household MPCs are adjusted to the U.S. level, which is substantially lower than the Peruvian level, via recalibration. In the counterfactual experiment, I find that excess consumption volatility disappears in most of the posterior distribution, including the posterior mean, median, and mode.

My paper is most closely related to [Guntin, Ottonello, and Perez \(2023\)](#). These two papers share a view that micro data, when interpreted through a heterogeneous-agent model, provide important information about what drives large consumption fluctuations. However, they come to different conclusions: [Guntin et al. \(2023\)](#) find that a trend shock drives large consumption swings, while I find that a financial friction shock and a stationary productivity shock mainly drive consumption fluctuations. Two key modeling differences lead the two papers to different conclusions. First, the precautionary saving stock is composed of liquid wealth in [Guntin et al. \(2023\)](#)'s model, while it is mostly composed of illiquid assets in my model. As a result, the consumption response to a trend shock is muted in my model because of financial friction and precautionary saving, both of which are strong because of the dominant presence of illiquid assets, while a trend shock can generate a large consumption response in [Guntin et al. \(2023\)](#)'s model because assets are entirely liquid. Second, [Guntin et al. \(2023\)](#) simulate a crisis by hitting the economy with a one-time, single-type shock, while I allow different types of shocks to be realized at different times. This flexibility allows my model to explain [Guntin et al. \(2023\)](#)'s main empirical finding in their micro

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<sup>2</sup>The financial friction shock is motivated by an observation that Peruvian households face volatile finance rates when they borrow. Under a relevant microfoundation, this observation can be interpreted as volatile fluctuations of a haircut on collateralized borrowing, which corresponds to the illiquid asset adjustment cost in the model. See section 4 for details.

<sup>3</sup>On the balanced growth path, most households (87.6%) cash out their illiquid assets. When financial friction is heightened, they fail to smooth consumption more significantly, as it becomes more costly to cash out their assets. Moreover, heightened financial friction makes households more concerned about a future low income path, enhancing their precautionary saving and thus further depressing their consumption.

moments while financial friction still plays a major role in generating consumption fluctuations.

More broadly, my paper is related to multiple strands of literature. First, there is rapidly growing literature examining how microlevel household behavior and its heterogeneity affect macroeconomic outcomes. Prominent works in this literature include [Kaplan et al. \(2018\)](#), [Auclert \(2019\)](#), [Krueger, Mitman, and Perri \(2016\)](#), [McKay, Nakamura, and Steinsson \(2016\)](#), [Auclert, Rognlie, and Straub \(2023\)](#), [Bayer, Luetticke, Pham-Dao, and Tjaden \(2019\)](#), and [Oh and Reis \(2012\)](#), among others. Many studies in this literature focus on the fact that even in advanced economies such as the U.S., a sizable fraction of households exhibit significantly higher MPC than what the PIH predicts. This paper contributes to this literature by exploiting a different margin: MPC is substantially higher in emerging economies than in developed economies. It finds that the difference in microlevel consumption behavior matters for aggregate dynamics to the extent that it explains one of the most salient patterns of emerging market business cycles, excess consumption volatility.

Second, there have been recent efforts to expand the first literature to open economies, such as [Auclert, Rognlie, Souchier, and Straub \(2021\)](#), [De Ferra, Mitman, and Romei \(2020\)](#), [De Ferra, Mitman, and Romei \(2023\)](#), [Druedahl, Ravn, Sunder-Plassmann, Sundram, and Waldstrøm \(2022\)](#), [Ferrante and Gornemann \(2022\)](#), [Guntin et al. \(2023\)](#), [Guo, Ottonello, and Perez \(2023\)](#), [Oskolkov \(2023\)](#), [Villalvazo \(2023\)](#), [Sunel \(2018\)](#), and [Zhou \(2022\)](#). My paper contributes to this literature by focusing on the implication of high MPC on emerging market business cycles.

Third, there is rich literature devoted to explaining emerging market business cycles, in which representative-agent models are dominantly used. Important examples include [Neumeyer and Perri \(2005\)](#), [Aguiar and Gopinath \(2007\)](#), [Uribe and Yue \(2006\)](#), [Garcia-Cicco, Pancrazi, and Uribe \(2010\)](#), [Chang and Fernández \(2013\)](#), [Chen and Crucini \(2016\)](#), and [Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe \(2011\)](#), among others. My paper contributes to this literature by bringing new intuitions and tools from the first literature regarding how household heterogeneity and microlevel behavior affect aggregate dynamics, applying them in the context of emerging market business cycles, and deriving new explanations.

Fourth, in terms of methodology, this paper has a commonality with [Bayer, Born, and Luetticke \(2023\)](#) and [Auclert, Rognlie, and Straub \(2020\)](#) in that Bayesian methods are applied to estimate a heterogeneous-agent model. Bayesian estimation requires a model to be solved many times. It only recently became possible to solve heterogeneous-agent models fast enough to conduct Bayesian estimation due to the development of new computational methods. The main contributors to this recent computational development include [Auclert, Bardóczy, Rognlie, and Straub \(2021\)](#), [Boppart, Krusell, and Mitman \(2018\)](#), [Ahn, Kaplan, Moll, Winberry, and Wolf \(2018\)](#), [Bayer and Luetticke \(2020\)](#), [Winberry \(2018\)](#), and [Reiter \(2009\)](#). Among the new methods, I use the one developed by [Auclert et al. \(2021\)](#).

The remainder of this paper is organized as follows. Section 2 specifies models. Section 3

takes the models to data in two steps, calibration and Bayesian estimation. Section 4 augments the HASOE model with a financial friction shock after presenting a motivating empirical observation. Section 5 conducts a counterfactual experiment in which MPCs are lowered to the U.S. level. Section 6 compares my paper and [Guntin et al. \(2023\)](#) in detail. Section 7 concludes.

## 2 Model

### 2.1 Heterogeneous-Agent Small Open Economy (HASOE) Model

I construct a HASOE model by incorporating [Kaplan et al. \(2018\)](#)'s two-asset household heterogeneity over liquid and illiquid assets into a standard emerging market business cycle model.<sup>4</sup> I adopt the two-asset heterogeneity for two reasons. First, it allows me to capture both realistically high MPCs and the correct amount of aggregate wealth. In a one-asset model, on the other hand, households must hold a small amount of assets to yield high MPCs.<sup>5</sup> This leads to an insufficient amount of aggregate capital, which is problematic for a business cycle analysis. Second, in reality, households in emerging economies put most of their savings in nonfinancial, illiquid assets ([Badarinza et al., 2019](#)), such as land, housing, and livestock. To reflect this fact, it is important to explicitly model illiquid assets, separately from liquid assets, as a saving vehicle that households can choose. Now, consider an economy composed of heterogeneous households and representative firms and banks.

**Working Households.** Almost all households (with fraction  $p$ ) work and earn labor income (workers hereafter). Workers face idiosyncratic earnings risk and trade liquid and illiquid assets. Liquid assets are bank deposits, and illiquid assets are firm shares. Compared to liquid assets, illiquid assets yield higher returns but are more expensive to trade. Workers exhibit idiosyncratic labor productivity, which can be decomposed into a component predictable with their observable characteristics ( $\Gamma$ ) and an unpredictable component bearing earnings risk ( $e$ ). Workers cannot take short positions in both liquid and illiquid assets. Each worker  $i$  solves the following problem.

$$\max_{\{c_{i,t}, b_{i,t}, a_{i,t}, v_{i,t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{i,t}^{1-\gamma}}{1-\gamma} \quad (1)$$

$$s.t. \quad c_{i,t} + b_{i,t} + v_{i,t} + \chi_t(v_{i,t}, a_{i,t-1}; \Gamma_i) = w_t \Gamma_i e_{i,t} \bar{l}_t + (1 - \xi)(1 + r_t^b) b_{i,t-1}, \quad (2)$$

$$v_{i,t} = a_{i,t} - (1 + r_t^a) a_{i,t-1}, \quad \text{and}$$

$$b_{i,t} \geq 0, \quad a_{i,t} \geq 0.$$

In the budget constraint (2),  $b_{i,t}$  and  $a_{i,t}$  are liquid and illiquid asset holdings, respectively, and  $(1 - \xi)(1 + r_t^b)$  and  $(1 + r_t^a)$  are their gross return rates. Since  $r^a = r^b$  on the balanced growth path

<sup>4</sup>However, I do not incorporate the nominal rigidity of [Kaplan et al. \(2018\)](#)'s model because my model is intended to be as close as possible to the conventional real models of emerging economies except for household heterogeneity.

<sup>5</sup>See [Kaplan and Violante \(2018\)](#) for a more detailed discussion.

and  $\xi > 0$ , illiquid assets yield higher returns than liquid assets.<sup>6</sup> To trade illiquid assets, workers must pay adjustment cost  $\chi_t(v_{i,t}, a_{i,t-1}; \Gamma_i)$ .

For the functional form of the illiquid asset adjustment cost, I closely follow [Auclert et al. \(2021\)](#)'s discrete-time version of [Kaplan et al. \(2018\)](#)'s model as follows.

$$\chi_t(v_{i,t}, a_{i,t-1}; \Gamma_i) = \chi_1 \left| \frac{v_{i,t}}{(1 + r_t^a)a_{i,t-1} + \chi_0 \Upsilon(\Gamma_i)X_{t-1}} \right|^{\chi_2} ((1 + r_t^a)a_{i,t-1} + \chi_0 \Upsilon(\Gamma_i)X_{t-1}),$$

where  $\chi_0 > 0$ ,  $\chi_1 > 0$ , and  $\chi_2 > 1$ .  $X_{t-1}$  is the stochastic trend of the economy, and  $\Upsilon(\Gamma_i)$  is the predictable component of earnings in a detrended steady state,  $E[w_t \Gamma_i e_{i,t} \bar{l}_t / X_{t-1} | \Gamma_i]$ .

Parameter  $\chi_1$  is the scaling factor for the adjustment cost and determines the overall importance of the adjustment cost in workers' optimization. When  $\chi_1$  increases, workers i) save more and ii) exhibit higher MPC for two reasons. First, workers who desire to smooth consumption by cashing out their assets fail to do so more significantly; thus, they save more (as they cash out assets less), and their consumption responds more strongly to a transitory income shock. Second, workers also have a stronger precautionary saving motive, which increases both saving and MPC, because they more intensely fear the realization of a low income path, as illiquid assets become more expensive to cash out.

Parameter  $\chi_2$  captures how less costly it is for wealthier households to adjust illiquid asset positions. When  $\chi_2 = 1$ , the adjustment cost becomes proportional to the absolute amount of illiquid asset position adjustment,  $v_{i,t}$ . As  $\chi_2$  increases above one, the adjustment cost becomes less costly for wealthier households, who have higher values of  $(1 + r_t^a)a_{i,t-1}$ . For this reason, parameter  $\chi_2$  is useful to make wealthy and poor households face different degrees of financial friction and thus exhibit different MPCs. Later, I calibrate  $\chi_1$  and  $\chi_2$  (jointly with  $\beta$ ) by targeting ten MPC moments over earnings deciles and workers' aggregate wealth.<sup>7</sup>

In the budget constraint (2), workers' earnings are  $w_t \Gamma_i e_{i,t} \bar{l}_t$ , where  $w_t$  is a wage rate per efficiency unit of labor,  $\Gamma_i$  and  $e_{i,t}$  are the predictable and unpredictable components of idiosyncratic labor productivity, respectively, and  $\bar{l}_t$  is a common labor supply determined by a labor union.<sup>8</sup>

<sup>6</sup>As we shall see later, liquid assets are bank deposits, and  $\xi(1 + r_t^b)$  is a deposit service fee that banks charge.

<sup>7</sup>What about  $\chi_0$ ? The term  $\chi_0 \Upsilon(\Gamma_i)X_{t-1}$  appears in the functional form of  $\chi_t(v_{i,t}, a_{i,t-1}; \Gamma_i)$  only to ensure that the denominator of  $[v_{i,t} / \{(1 + r_t^a)a_{i,t-1} + \chi_0 \Upsilon(\Gamma_i)X_{t-1}\}]$  is nonzero. (Reflecting its purpose, in calibration, I assign an arbitrary small number, 0.01, to  $\chi_0$ .) In this term,  $\chi_0$  is augmented with  $\Upsilon(\Gamma_i)X_{t-1}$  to make workers' problem i) stationary after detrending and ii) identical across different  $\Gamma_i$ 's after normalization. See Online Appendix A.2.

<sup>8</sup>In my HASOE model, I do not let individual workers choose their labor supply. Instead, I delegate the decision to a labor union and write its optimization problem such that the labor supply equation coincides with that of the corresponding RASOE model that we shall see later. The reason is as follows: when individual workers determine their labor supply under widely used preference specifications, the model exhibits counterfactual patterns in important dimensions. Under separable labor disutility, aggregate labor supply declines substantially during booms because of the wealth effect. This problem is common in macroeconomic models of emerging economies (including those with representative households) because they are designed to exhibit large consumption fluctuations, which also generate a large wealth effect. For this reason, macroeconomic models of emerging economies usually impose the preference



The labor union makes a labor supply decision by linearly weighting the aggregate labor income  $w_t L_t$  and labor disutility  $X_{t-1} \frac{1}{1+\omega} \bar{l}_t^{1+\omega}$  as follows.

$$\begin{aligned} \max_{\bar{l}_t, L_t} \quad & w_t L_t - \kappa \left( X_{t-1} \frac{1}{1+\omega} \bar{l}_t^{1+\omega} \right) \\ \text{s.t.} \quad & L_t = p \bar{\Gamma} \bar{e} \bar{l}_t, \end{aligned} \quad (3)$$

where  $\kappa > 0$ ,  $L_t$  is aggregate labor supply (in efficiency units), and  $\bar{\Gamma} (:= E[\Gamma_i])$  and  $\bar{e} (:= E[e_{i,t}])$  are the cross-sectional average of the predictable and unpredictable components of workers' idiosyncratic productivity, respectively. As a result of the optimization, the labor supply is determined by

$$w_t (p \bar{\Gamma} \bar{e})^{1+\omega} = \kappa X_{t-1} L_t^\omega, \quad t \geq 0. \quad (4)$$

The predictable component of idiosyncratic labor productivity,  $\Gamma_i$ , follows a lognormal distribution:  $\log \Gamma_i \sim N(0, \sigma_\Gamma)$ . The unpredictable component of idiosyncratic labor productivity,  $e_{i,t}$ , is composed of a persistent component ( $e_{1,i,t}$ ) and a transitory component ( $e_{2,i,t}$ ) as follows.<sup>9</sup>

$$\begin{aligned} \log e_{i,t} &= \log e_{1,i,t} + \log e_{2,i,t}, \\ \log e_{1,i,t} &= \rho_{e_1} \log e_{1,i,t-1} + \varepsilon_{1,i,t}, \quad \varepsilon_{1,i,t} \sim N(0, \sigma_{\varepsilon_1}^2), \quad \text{and} \\ \log e_{2,i,t} &= \varepsilon_{2,i,t}, \quad \varepsilon_{2,i,t} \sim N(0, \sigma_{\varepsilon_2}^2). \end{aligned}$$

Let  $G(\Gamma)$  denote the cumulative distribution function (CDF) of  $\Gamma$ , and  $\Psi_t(e_1, e_2, b_-, a_- | \Gamma)$  denote the CDF of  $(e_1, e_2, b_-, a_-)$  conditional on  $\Gamma$  in period  $t$  among workers. Let  $c_t(e_1, e_2, b_-, a_-; \Gamma)$ ,  $b_t(e_1, e_2, b_-, a_-; \Gamma)$ , and  $a_t(e_1, e_2, b_-, a_-; \Gamma)$  denote the policy functions of workers with  $\Gamma$  in period  $t$ .<sup>10</sup> The law of motion for  $\Psi_t(e_1, e_2, b_-, a_- | \Gamma)$  is determined as follows.

$$\begin{aligned} \Psi_{t+1}(e'_1, e'_2, b, a | \Gamma) &= \int_{e_1, e_2, b_-, a_-} [P(e_{1,t+1} \leq e'_1 | e_{1,t} = e_1) P(e_{2,t+1} \leq e'_2) \\ &\quad I_{\{b_t(e_1, e_2, b_-, a_-; \Gamma) \leq b, a_t(e_1, e_2, b_-, a_-; \Gamma) \leq a\}}(e_1, e_2, b_-, a_-)] d\Psi_t(e_1, e_2, b_-, a_- | \Gamma), \end{aligned} \quad (5)$$

devised by [Greenwood, Hercowitz, and Huffman \(1988\)](#) (GHH preference hereafter) because it eliminates the wealth effect. In my HASOE model, however, another counterfactual pattern emerges under the GHH preference: workers exhibit abnormally high MPC compared to the data. This is because workers try to smooth  $(c - h(l))$  rather than  $c$ , where  $h(l)$  is labor disutility. Thus, consumption comoves too strongly with earnings, yielding excessively high MPC.

In the heterogeneous-agent New Keynesian (HANK) literature, researchers also find that models exhibit counterfactual patterns when individual households determine their labor supply, although in different dimensions. Some researchers prefer to circumvent these problems by introducing a labor union to which the labor supply decision is delegated. (See [Auclert, Bardóczy, and Rognlie \(2021\)](#) for a detailed discussion.) In the same spirit, I introduce a labor union to circumvent the problems caused by individual labor supply decisions.

<sup>9</sup>The labor productivity process specification in the model is consistent with the income process specification imposed in the MPC estimation. See Online Appendix E.1.

<sup>10</sup> I attach the time subscript to the policy functions because they depend on the state vector  $\mathbf{S}_t$ , which includes conditional distribution  $\Psi_t(e_1, e_2, b_-, a_- | \Gamma)$ , stochastic trend  $X_{t-1}$ , and other predetermined and exogenous variables in the economy. See footnote 13 for details on the state vector  $\mathbf{S}_t$ .

where  $I_{\{X\}}(x)$  is an indicator function (*i.e.*,  $I_{\{X\}}(x) = 1$  if  $x \in X$ , 0 otherwise).

**Entrepreneurial Households** A tiny fraction  $(1 - p)$  of households do not work and earn income only from the returns on firm share holdings and pure rents from banks (entrepreneurs hereafter). Unlike workers, entrepreneurs do not face idiosyncratic earnings risk and do not pay the adjustment cost when trading firm shares. Entrepreneurs solve the following optimization problem.

$$\begin{aligned} \max_{\{C_t^E, A_t^E\}_{t=0}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} (\beta_E)^t \frac{(C_t^E)^{1-\gamma}}{1-\gamma} \\ \text{s.t.} \quad & C_t^E + A_t^E = R_t^E + (1 + r_t^a)A_{t-1}^E, \quad \text{and} \\ & \lim_{j \rightarrow \infty} E_t \left[ A_{t+j}^E / \left( \prod_{s=1}^j (1 + r_{t+s}^a) \right) \right] \geq 0, \end{aligned} \quad (6)$$

where  $C_t^E$ ,  $A_t^E$ , and  $R_t^E$  are their consumption, shareholdings, and pure rents, respectively.

I introduce entrepreneurs into my model for two reasons. First and foremost, in reality, a tiny fraction of the richest households hold a significant fraction of wealth, and this top wealth share is unlikely to be accumulated by a precautionary saving motive. By introducing entrepreneurs, the top wealth share is excluded from the aggregate precautionary saving stock in the model (as it is held by entrepreneurs).<sup>11</sup> Second, as [Bayer et al. \(2019\)](#) note, the pure rents of the economy can be allocated back to households without distorting factor returns or introducing a new asset.

**Aggregation.** Let  $C_t$ ,  $A_t$ ,  $B_t$ , and  $\chi_t^{agg}$  denote the aggregate consumption, firm value, deposit, and illiquid asset adjustment cost, respectively, and  $C_t^W$ ,  $A_t^W$ ,  $B_t^W$ , and  $\chi_t^W$  denote the cross-sectional average of workers' consumption, illiquid asset holdings, liquid asset holdings, and illiquid asset adjustment cost, respectively. The aggregate quantities are constructed as follows.

$$C_t = pC_t^W + (1 - p)C_t^E, \quad C_t^W = \int_{\Gamma} \int_{e_1, e_2, b_-, a_-} c_t(e_1, e_2, b_-, a_-; \Gamma) d\Psi_t dG. \quad (7)$$

$$A_t = pA_t^W + (1 - p)A_t^E, \quad A_t^W = \int_{\Gamma} \int_{e_1, e_2, b_-, a_-} a_t(e_1, e_2, b_-, a_-; \Gamma) d\Psi_t dG. \quad (8)$$

$$B_t = pB_t^W, \quad B_t^W = \int_{\Gamma} \int_{e_1, e_2, b_-, a_-} b_t(e_1, e_2, b_-, a_-; \Gamma) d\Psi_t dG. \quad (9)$$

$$\chi_t^{agg} = p\chi_t^W, \quad \chi_t^W = \int_{\Gamma} \int_{e_1, e_2, b_-, a_-} \chi_t(a_t(e_1, e_2, b_-, a_-; \Gamma) - (1 + r_t^a)a_-, a_-; \Gamma) d\Psi_t dG. \quad (10)$$

**Firms.** Competitive firms produce output  $Y_t$  using capital  $K_{t-1}$  and labor  $L_t$ , make investment  $I_t$ ,

<sup>11</sup>As we shall see later, workers' strong precautionary saving plays several important roles in the main results of this paper. Without having entrepreneurs in the model, the strength of precautionary saving behavior can be overrated.



and borrow funds  $F_t$  from domestic banks. They solve the following optimization problem.

$$\begin{aligned}
& \max_{\{K_t, F_t, L_t, Y_t, I_t, \Pi_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} Q_{0,t} \Pi_t \tag{11} \\
s.t. \quad & \Pi_t = Y_t - w_t L_t - I_t - \Phi(K_t, K_{t-1}) + F_t - (1 + r_{t-1}) F_{t-1}, \\
& Y_t = z_t K_{t-1}^{\alpha} (X_t L_t)^{1-\alpha}, \\
& I_t = K_t - (1 - \delta) K_{t-1}, \\
& \Phi(K_t, K_{t-1}) = \frac{\phi}{2} \left( \frac{K_t}{K_{t-1}} - g^* \right)^2 K_{t-1}, \\
& Q_{0,t} = \begin{cases} 1 & \text{if } t = 0, \\ 1 / (\prod_{s=1}^t (1 + r_s^a)) & \text{if } t \geq 1, \end{cases} \quad \text{and} \\
& \lim_{j \rightarrow \infty} E_t \left[ F_{t+j} / \left( \prod_{s=1}^j (1 + r_{t+s}^a) \right) \right] \leq 0,
\end{aligned}$$

where  $\Pi_t$  is the per-period profit,  $\Phi(K_t, K_{t-1})$  is an adjustment cost for capital accumulation,  $z_t$  is the stationary component of firms' productivity, and  $X_t$  is the nonstationary component (or stochastic trend) of firms' productivity. Firms discount profit flows using return rates on their shares. As we shall see below, the firms' objective function is the total value of the firms.

**Asset Return and Price.** Let  $s_{it}$  and  $s_t^E$  be worker  $i$ 's and a representative entrepreneur's firm shares, respectively, when the total shares are normalized to 1. Let  $q_t$  be the price of the shares after the current profits are distributed as dividends. Since total shares are normalized to 1,  $q_t$  also represents the total value of the firms after distributing current profits. By construction, we have

$$\begin{aligned}
a_{it} &= s_{it} q_t, \quad (1 + r_t^a) a_{i,t-1} = s_{i,t-1} (\Pi_t + q_t), \quad t \geq 0, \quad \text{and} \\
A_t^E &= s_t^E q_t, \quad (1 + r_t^a) A_{t-1}^E = s_{t-1}^E (\Pi_t + q_t), \quad t \geq 0.
\end{aligned}$$

Combining these two equations with aggregation equation (8), we can obtain

$$A_t = q_t, \quad t \geq 0, \quad \text{and} \tag{12}$$

$$1 + r_t^a = (\Pi_t + q_t) / q_{t-1}, \quad t \geq 0. \tag{13}$$

By iterating equation (13) forward to solve  $q_0$  and taking an expectation, we can verify that the firms' objective function  $E_0 \sum_{t=0}^{\infty} Q_{0,t} \Pi_t$  is equal to  $\Pi_0 + q_0$ . In other words, firms maximize their total value before distributing current profits. This explains why firms discount profit flows with asset returns in their optimization.

It is worth noting how  $\{r_t^a\}_{t=0}^{\infty}$  are determined in equilibrium. From period 1 onward,  $\{r_t^a\}_{t=1}^{\infty}$

are subject to the following optimality condition of firms.

$$E_t[(1+r_t)/(1+r_{t+1}^a)] = 1, \quad t \geq 0. \quad (14)$$

When we consider impulse responses to an MIT shock (*i.e.*, without aggregate uncertainty), this equation becomes  $r_{t+1}^a = r_t$ ,  $t \geq 0$ . On the other hand, the return in period 0,  $r_0^a$ , is not determined by equation (14). Instead,  $r_0^a$  is solely determined by  $\Pi_0$ ,  $q_0$ , and  $q_{-1}$  through equation (13).

**Banks.** Banks play a passive role in the model. They lend funds  $F_t$  to firms and finance the funds by (i) issuing debt  $D_t$  in the international financial market and (ii) intermediating household deposits,  $B_t$ . These funds should be balanced each period:

$$F_t = D_t + B_t, \quad t \geq 0. \quad (15)$$

The gross rate of banks' financing cost through the intermediation of household deposit  $B_{t-1}$  is  $(1+r_t^b)$ . This financing cost is composed of a gross return on household deposits  $(1-\xi)(1+r_t^b)$  and a service charge  $\xi(1+r_t^b)$ . Banks can frictionlessly adjust the financing sources, and thus, the financing cost is equalized between the two sources, household deposits and international debt:

$$1+r_t^b = 1+r_{t-1}, \quad t \geq 0. \quad (16)$$

Since banks are competitive, they charge an interest rate on  $F_t$  that equals the financing cost,  $r_t$ .

In addition, banks facilitate trades in firm shares among workers and earn facilitation fees  $\chi_t^{agg}$ . While carrying out their roles, banks create pure rents,  $\xi(1+r_t^b)B_{t-1}$  and  $\chi_t^{agg}$ . As discussed above, entrepreneurs hold the right to claim these pure rents:

$$(1-p)R_t^E = \xi(1+r_t^b)B_{t-1} + \chi_t^{agg}. \quad (17)$$

**International Financial Market.** The interest rate  $r_t$  in the international financial market is specified as follows.

$$r_t = r^* + \psi \left\{ \exp \left( \frac{\bar{D}_t/X_t - \bar{D}^*}{\bar{Y}^*} \right) - 1 \right\} - \theta_z(z_t - 1) - \theta_g \left( \frac{g_t}{g^*} - 1 \right) + \mu_t - 1, \quad t \geq 0, \quad (18)$$

where  $\psi > 0$ ,  $\theta_z > 0$ , and  $\theta_g > 0$ .  $\bar{D}_t$  is the cross-sectional average of banks' international debt. Individual banks and firms regard  $\bar{D}_t$  as exogenous, but in equilibrium, individual banks' international debt  $D_t$  is equal to  $\bar{D}_t$ .  $\bar{D}^*$ ,  $\bar{Y}^*$ ,  $g^*$ , and  $r^*$  are the long-run averages of  $\bar{D}_t/X_t$ ,  $Y_t/X_{t-1}$ ,  $g_t := X_t/X_{t-1}$ , and  $r_t$ , respectively, and  $\mu_t$  is an exogenous disturbance to the interest rate.<sup>12</sup>

<sup>12</sup>A reduced-form specification of the interest rate in the international financial market, such as equation (18),

**Aggregate Shocks.** Three aggregate shocks hit the economy: a stationary productivity shock  $z_t$ , a trend shock  $g_t$ , and an interest rate shock  $\mu_t$ . I assume that each shock follows an AR(1) process:

$$\begin{aligned}\log z_t &= \rho_z \log z_{t-1} + \varepsilon_t^z, & \varepsilon_t^z &\sim N(0, \sigma_z^2), \\ \log(g_t/g^*) &= \rho_g \log(g_{t-1}/g^*) + \varepsilon_t^g, & \varepsilon_t^g &\sim N(0, \sigma_g^2), \quad \text{and} \\ \log \mu_t &= \rho_\mu \log \mu_{t-1} + \varepsilon_t^\mu, & \varepsilon_t^\mu &\sim N(0, \sigma_\mu^2).\end{aligned}\tag{19}$$

**Trade Balance.** The trade balance of the economy,  $TB_t$ , is determined as follows.

$$TB_t = -D_t + (1 + r_{t-1})D_{t-1}, \quad t \geq 0.\tag{20}$$

**Equilibrium.** Given the initial conditions on  $\Psi_0(e_1, e_2, b_-, a_- | \Gamma)$ ,  $X_{-1}$ ,  $A_{-1}$ ,  $A_{-1}^E$ ,  $K_{-1}$ ,  $D_{-1}$ ,  $B_{-1}$ ,  $F_{-1}$ , and  $r_{-1}$ ,<sup>13</sup> (i) individual workers' policy functions  $\{c_t(e_1, e_2, b_-, a_-; \Gamma), b_t(e_1, e_2, b_-, a_-; \Gamma), a_t(e_1, e_2, b_-, a_-; \Gamma)\}_{t=0}^\infty$  that solve workers' problem (1), (ii) conditional cumulative distributions  $\{\Psi_t(e_1, e_2, b_-, a_- | \Gamma)\}_{t=1}^\infty$  that evolve over time according to equation (5), (iii) prices and aggregate variables  $\{r_t^b, r_t^a, r_t, w_t, q_t, \bar{l}_t, L_t, \Pi_t, Y_t, I_t, K_t, F_t, D_t, TB_t, C_t, C_t^E, A_t, A_t^E, R_t^E, B_t, \chi_t^{agg}\}_{t=0}^\infty$  satisfying the optimality conditions for entrepreneurs' problem (6), the optimality conditions for firms' problem (11), aggregation equations (7) - (10), and other equilibrium conditions (3), (4), (12), (13), (15) - (18), and (20), and (iv) aggregate shocks  $\{z_t, g_t, \mu_t\}_{t=0}^\infty$  that follow the stochastic processes specified in equation (19) constitute the equilibrium of the economy.

## 2.2 Representative-Agent Small Open Economy (RASOE) Model

One of the goals of this paper is to compare the ways in which RASOE and HASOE models explain emerging market business cycles. To this end, I consider a conventional RASOE model in the literature (Neumeyer and Perri (2005), Aguiar and Gopinath (2007), Garcia-Cicco et al. (2010), and Chang and Fernández (2013)), which can be recovered from the HASOE model presented above by simply replacing heterogeneous households with representative households.<sup>14</sup> The representative households trade assets  $A_t$ , which are firm shares, and supply labor  $L_t$ . They optimize

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is widely used in emerging market business cycle studies, particularly when models are intended to be first-order approximated with respect to aggregate shocks. (See, for instance, Neumeyer and Perri (2005), Garcia-Cicco et al. (2010), and Chang and Fernández (2013).) In equation (18), interest rates are higher when the international debt  $\bar{D}_t$  is larger and the productivities  $z_t$  and  $g_t$  are lower. In this respect, equation (18) reflects the theoretical implication of sovereign default models such as Arellano (2008) and Mendoza and Yue (2012) in a reduced-form manner.

<sup>13</sup>The initial conditions given here specify the predetermined objects in state vector  $\mathbf{S}_0$ . Referring back to footnote 10, state vector  $\mathbf{S}_t$  is composed of predetermined objects  $\Psi_t(e_1, e_2, b_-, a_- | \Gamma)$ ,  $X_{t-1}$ ,  $A_{t-1}$ ,  $A_{t-1}^E$ ,  $K_{t-1}$ ,  $D_{t-1}$ ,  $B_{t-1}$ ,  $F_{t-1}$ , and  $r_{t-1}$  and aggregate exogenous variables  $z_t$ ,  $g_t$ , and  $\mu_t$ .

<sup>14</sup>The consequent RASOE model features a decentralized economy. In Online Appendix B.1, I present an equivalent, centralized version of the RASOE model, which appears far more frequently in related studies.

the GHH preference subject to budget constraints and the no-Ponzi-game constraint as follows.

$$\begin{aligned} \max_{\{C_t, A_t, L_t\}_{t=0}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} (\beta_R)^t \frac{(C_t - \kappa_R X_{t-1} L_t^{1+\omega})^{1-\gamma}}{1-\gamma} \\ \text{s.t.} \quad & C_t + A_t = w_t L_t + (1 + r_t^a) A_{t-1}, \quad t \geq 0, \text{ and} \\ & \lim_{j \rightarrow \infty} E_t \left[ A_{t+j} \middle/ \left( \prod_{s=1}^j (1 + r_{t+s}^a) \right) \right] \geq 0, \end{aligned}$$

where  $C_t$  is consumption,  $w_t$  is wage,  $(1 + r_t^a)$  is a gross return on  $A_{t-1}$ , and  $X_{t-1}$  is a stochastic trend of the economy. By imposing the GHH preference, the wealth effect is removed in the labor supply decision, as  $L_t$  is determined by  $w_t = (1 + \omega) \kappa_R X_{t-1} L_t^\omega$ .<sup>15</sup> This labor supply equation is identical to that in the HASOE model under  $(1 + \omega) \kappa_R = \frac{\kappa}{(p\bar{\Gamma}\bar{e})^{1+\omega}}$ . In other words, the HASOE model does not deviate from the RASOE model in the dimension of aggregate labor supply.

The remaining parts of the economy are identical to the corresponding parts of the HASOE model. Firms solve problem (11). Banks balance funds using equation (15) with  $B_t = 0$  (because representative households do not save in deposits). Asset return  $r_t^a$  and price  $q_t$  again satisfy equations (12) and (13). The interest rate  $r_t$  in the international financial market is determined by equation (18). Three aggregate shocks,  $z_t$ ,  $g_t$ , and  $\mu_t$ , hit the economy and are assumed to follow an AR(1) process, as specified in equation (19). Trade balance is determined by equation (20). In Online Appendix B.1, I present the complete set of equilibrium conditions.

### 2.3 Solving the Models

To study business cycles using Bayesian estimation, a model needs to be solved fast enough. There is well-established literature on the Bayesian estimation of representative-agent models, but for heterogeneous-agent models, it only recently became possible to solve these models fast enough to conduct Bayesian estimation due to the development of new solution methods.<sup>16</sup>

Among the new methods, I adopt Auclert et al. (2021)'s method, which computes linearized aggregate dynamics based on Boppart et al. (2018)'s finding that the MA( $\infty$ ) representation of a linearized model regarding aggregate uncertainty can be fully recovered from impulse responses to an MIT shock due to certainty equivalence. Since this method exploits impulse responses to an MIT shock, in Online Appendices A.1 and B.1, I characterize the equilibrium of the HASOE and RASOE models, respectively, when the economy is subject to deterministic paths of aggregate shocks,  $\{z_t, g_t, \mu_t\}_{t=0}^{\infty}$ .

<sup>15</sup>The wealth effect removal is an important reason why the GHH preference is common in emerging market business cycle models, as discussed in footnote 8.

<sup>16</sup>The main contributors to this recent development include Auclert et al. (2021), Boppart et al. (2018), Ahn et al. (2018), Bayer and Luetticke (2020), Winberry (2018), and Reiter (2009).

Both my HASOE and RASOE models exhibit a stochastic trend, and thus, the equilibrium needs to be detrended to become stationary. In Online Appendices A.2 and B.2, I present a detrended stationary equilibrium of the HASOE and RASOE models, respectively. Then, Online Appendix C discusses how to solve the detrended equilibrium using Auclert et al. (2021)’s method.

### 3 Taking Models to Data

Both the HASOE and RASOE models are fitted to the Peruvian economy. Specifically, I take the models to Peruvian data through two steps. First, I calibrate parameters determining key moments on the balanced growth path. Second, I estimate parameters governing aggregate dynamics around the balanced growth path using Bayesian methods. This two-step procedure is possible because the parameters estimated in the second step do not affect the balanced growth path.

#### 3.1 Calibration

In both the HASOE and RASOE models, the time unit is a quarter. Table 1 reports the calibrated parameter values and target moments or information sources used for the calibration. For parameters  $g^*$ ,  $r^*$ ,  $\alpha$ ,  $\delta$ ,  $\tilde{D}^*$ ,  $\gamma$ , and  $\omega$ , I assign the same parameter values to the HASOE and RASOE models. Parameters  $g^*$  and  $r^*$  are calibrated to the long-run average output growth rate and real lending interest rate in data, respectively. Parameters  $\alpha$ ,  $\delta$ , and  $\tilde{D}^*$  are calibrated by matching the long-run average capital-output ratio (10.91), investment-output ratio (0.191), and trade-balance-to-output ratio (0.043) from data in the related equilibrium conditions.<sup>17,18</sup> Parameters  $\gamma$  and  $\omega$  are assigned the values used in Garcia-Cicco et al. (2010), which are common in the related literature.

Parameters  $\sigma_\Gamma$ ,  $\rho_{e1}$ ,  $\sigma_{e1}$ ,  $\sigma_{e2}$ ,  $\beta$ ,  $\chi_1$ ,  $\chi_2$ ,  $\chi_0$ ,  $\kappa$ ,  $\xi$ ,  $\beta_E$ , and  $p$  appear only in the HASOE model.

<sup>17</sup>The output growth, investment-output ratio, and trade-balance-to-output ratio are computed using Banco Central de Reserva del Perú (BCRP)’s national account data from 1980–2018. (Specifically, real quarterly national account data are obtained, seasonally adjusted, and transformed into a per capita term.) The capital-output ratio is computed using Feenstra, Inklaar, and Timmer (2015)’s Penn World Table (version 9.1) capital and output data from 1980–2017. The real lending interest rates are computed by deflating BCRP’s data on lending rates in foreign currency (TAMEX) from 1992–2018 with the expected inflation on U.S. CPIs. The expected inflation is constructed by taking an average inflation rate over the current and past three quarters, following Neumeyer and Perri (2005) and Uribe and Yue (2006). (Atkeson and Ohanian (2001) empirically support this approximation of expected inflation.)

<sup>18</sup>In the literature, interest rates are often constructed by adding J.P. Morgan’s EMBIG sovereign bond spreads to U.S. interest rates (‘EMBIG interest rates’ hereafter). Instead, I construct interest rates based on BCRP data series (‘BCRP interest rates’ hereafter). I find that these two interest rates are highly correlated (correlation 0.863), but their means are substantially different; the average unannualized quarterly EMBIG interest rate is 0.007, while that of the BCRP interest rate is 0.021. Given that the long-run average value of  $TB_t/Y_t$  is calibrated to its data counterpart, there is a one-to-one relationship between  $r^*$  and  $D_t/Y_t$  on the balanced growth path through an equilibrium condition  $r^* = \frac{TB_t/Y_t}{D_t/Y_t} g^* + (g^* - 1)$ . Using this equation, I recover the value of  $r^*$  that corresponds to the long-run average value of  $D_t/Y_t$  in Milesi-Ferretti and Lane (2017)’s dataset. The value of such  $r^*$  is 0.025, which is far closer to the average BCRP rate than the average EMBIG rate. Based on this observation, I use BCRP interest rates rather than EMBIG interest rates so that the model generates  $D_t/Y_t$  close to Milesi-Ferretti and Lane (2017)’s debt data.

Table 1: Calibration for the Peruvian Economy

Description		Value	Target / Source
<i>Common in HASOE and RASOE</i>			
$g^*$	long-run average gross growth rate	1.004	$E[Y_t/Y_{t-1}]$
$r^*$	long-run average lending rate	0.021	BCRP, U.S. CPI
$\alpha$	capital income share	0.378	$(K/Y)(r^* + \delta)/g^*$
$\delta$	depreciation rate	0.014	$g^*(I/Y)/(K/Y) - (g^* - 1)$
$\tilde{D}^*$	international debt	10.57	$(TB/Y) \hat{Y}^*/(1 + r^* - g^*)$
$\gamma$	inverse of IES	2.000	<a href="#">Garcia-Cicco et al. (2010)</a>
$\omega$	inverse of labor supply elasticity	0.600	<a href="#">Garcia-Cicco et al. (2010)</a>
<i>HASOE only - earnings process</i>			
$\sigma_\Gamma$	S.D. of the predictable component	0.656	} ENAHO
$\rho_{e_1}$	persistence of the unpredictable, AR(1) component	0.968	
$\sigma_{e_1}$	S.D. of shocks to the unpredictable, AR(1) component	0.134	
$\sigma_{e_2}$	S.D. of shocks to the unpredictable, <i>i.i.d.</i> component	0.464	
<i>HASOE only - targeting MPCs &amp; Workers' Aggregate Wealth</i>			
$\beta$	workers' discount factor	0.948	} MPC estimates & earnings-income ratio in ENAHO
$\chi_1$	scale parameter for illiquid asset adjustment cost	6.694	
$\chi_2$	convexity parameter for illiquid asset adjustment cost	1.724	
$\chi_0$	non-zero denominator in illiquid asset adjustment cost	0.010	
<i>HASOE only - other parameters</i>			
$\kappa$	scale parameter for labor disutility	5.449	$L = 1$ on the b.g.p
$\xi$	long-run average spread	0.020	BCRP, U.S. CPI
$\beta_E$	entrepreneurs' discount factor	0.987	$(g^*)^\gamma/(1 + r^*)$
$p$	share of workers (= 1 – share of entrepreneurs)	0.987	WID
<i>RASOE only</i>			
$\beta_R$	representative households' discount factor	0.987	$(g^*)^\gamma/(1 + r^*)$
$\kappa_R$	scale parameter for labor disutility	1.660	$L = 1$ on the b.g.p

Notes: The time unit is a quarter. 'b.g.p' in the 'Target/Source' column represents the balanced growth path.

Parameters  $\sigma_\Gamma$ ,  $\rho_{e_1}$ ,  $\sigma_{e_1}$ , and  $\sigma_{e_2}$  govern workers' earnings process. I calibrate them using earnings data from the 2011–2018 waves of a nationally representative Peruvian household survey, Encuesta Nacional de Hogares (ENAHO). I first remove predictable components from household earnings using observable characteristics. Then, I calibrate  $\sigma_\Gamma$  using the predictable components and  $\rho_{e_1}$ ,  $\sigma_{e_1}$ , and  $\sigma_{e_2}$  by applying Floden and Lindé (2001)'s method to the residual components.<sup>19</sup>

Parameters  $\beta$ ,  $\chi_1$ , and  $\chi_2$  are calibrated by targeting MPC moments and workers' aggregate wealth.<sup>20</sup> For the target MPC moments, I estimate the Peruvian quarterly MPC at each resid-

<sup>19</sup>See Online Appendix E.1 for details.

<sup>20</sup>For  $\chi_0$ , I assign an arbitrary small number, 0.01, as discussed in footnote 7.



ual earnings ( $e_{i,t}$ ) decile by applying [Blundell et al. \(2008\)](#)'s method to the ENAHO data, as in [Hong \(2023\)](#).<sup>21,22</sup> To target the correct amount of workers' aggregate wealth, I match the earnings-income ratio (or, equivalently, (labor income) / (labor income + capital income)) on the balanced growth path of the model with the ratio in the ENAHO data, 0.817.<sup>23</sup> I implement this joint calibration by minimizing the following objective function  $J$ :

$$J = \varpi \left\{ \frac{w_t \bar{\Gamma} \bar{e}_t}{w_t \bar{\Gamma} \bar{e}_t + [r_t^a A_{t-1}^W + \{(1 - \xi)(1 + r_t^b) - 1\} B_{t-1}^W]} \text{ on the balanced growth path} - 0.817 \right\}^2 \\ + (1 - \varpi) \left\{ (MPC_{model} - MPC_{data}) \cdot V_{mpc} \cdot (MPC_{model} - MPC_{data})' \right\},$$

where  $\varpi$  denotes a weight on the first target (workers' earnings-income ratio),  $MPC_{model}$  and  $MPC_{data}$  denote a 10-by-1 vector of the model-predicted MPC on the balanced growth path and the estimated MPC at each earnings decile, respectively, and  $V_{mpc}$  is a weight matrix, which I choose to be a diagonal matrix whose diagonal elements equal the earnings share of each decile.

The joint calibration matches targets well, even though only three parameters are used to target eleven moments. First, workers' earnings-income ratio is 0.832 in the model, which is close to its data counterpart, 0.817. Second, [Figure 1](#) plots the model-predicted MPC (labeled 'Model') and the estimated MPC (labeled 'Data') at each earnings decile and shows that the former tracks the latter closely. Importantly, the data strongly suggest that households significantly deviate from the PIH, as the mean quarterly MPC estimate across deciles (0.209) is substantially greater than zero, and the model successfully captures such deviation by matching the MPC moments.

To gauge how large the financial friction is, I compute the ratio between workers' total adjustment cost and adjustment size ( $\chi_t^W / \int_{\Gamma} \int_{e_1, e_2, b_-, a_-} |v_{i,t}| d\Psi_t dG$ ). The ratio is 43.5%.<sup>24</sup>

The rest of the HASOE-specific parameters ( $\kappa$ ,  $\xi$ ,  $\beta_E$ , and  $p$ ) are calibrated as follows. Parameter  $\kappa$  is calibrated such that the aggregate labor supply is normalized to 1 on the balanced growth

<sup>21</sup>In estimating and targeting MPCs, observations are grouped by residual earnings  $e_{i,t}$  (or, equivalently, the unpredictable component of earnings) instead of total earnings ( $w_t \Gamma_i e_{i,t} l_t$ ) because  $e_{i,t}$  bears risk and thus induces precautionary saving and MPC heterogeneity, while  $\Gamma_i$  does not.

<sup>22</sup>Compared to [Hong \(2023\)](#), the consumption measure is changed from nondurable consumption to total consumption (including durable consumption) to be consistent with the aggregate consumption measure. The sample period is also changed to 2011–2018. Some of the early waves (2004–2010) used in [Hong \(2023\)](#) are not used here because quarterly expenses of some key durable goods are unavailable in these waves. Online Appendix E provides further details of the MPC estimation and data processing procedures.

<sup>23</sup>In ENAHO, labor and capital incomes are not distinguishable within self-employment income. As in [Diaz-Gimenez, Quadrini, and Rios-Rull \(1997\)](#), [Krueger and Perri \(2006\)](#), and [Hong \(2023\)](#), I split self-employment income into labor and capital income parts using the ratio between unambiguous labor and capital incomes. In ENAHO, the ratio of (unambiguous labor income) / (unambiguous labor income + unambiguous capital income) is 0.817, and it becomes the earnings-income ratio once self-employment income is split according to this ratio. This ratio is close to the ratio that [Diaz-Gimenez et al. \(1997\)](#) and [Krueger and Perri \(2006\)](#) use for their U.S. sample, 0.864.

<sup>24</sup>As we shall see in section 4, this ratio can be interpreted as an average haircut rate for collateralized borrowing, under which households collateralize  $|v_{i,t}|$  amount of illiquid assets and cash out  $|v_{i,t}| - \chi_t(v_{i,t})$ , with haircut  $\chi_t(v_{i,t})$ .

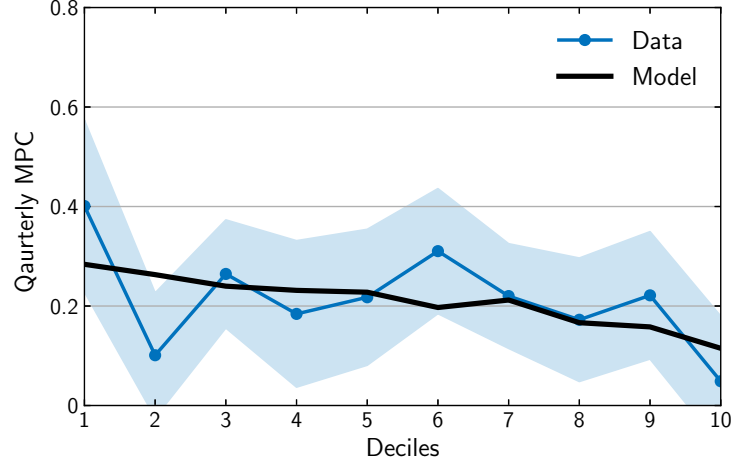


Figure 1: Quarterly MPCs in Peru: Data vs. Model

*Notes:* This figure plots the model-predicted MPC (labeled ‘Model’) and the estimated MPC (labeled ‘Data’) at each earnings decile. Integer 1 on the  $x$ -axis denotes the bottom decile. Shaded areas represent 95% confidence intervals.

path. Parameter  $\xi$  is calibrated to a spread between long-run average lending and deposit interest rates in data.<sup>25</sup> Parameter  $\beta_E$  is calibrated such that entrepreneurs’ Euler equation holds on the balanced growth path. Parameter  $p$  is calibrated such that entrepreneurs’ wealth share matches the top  $100(1 - p)\%$  share of wealth in data.<sup>26</sup>

Among the calibration targets used for the HASOE model, only three are distributional moments: the top 1.3% ( $= 100(1 - p)\%$ ) share of wealth and the standard deviations of the predictable and unpredictable earnings components. The model, however, predicts many more distributional moments than the three targets. Table 2 compares some key moments for wealth, earnings, and consumption distributions between the model and data. All the moments compared in the table are untargeted except for the top 1.3% wealth share. Table 2 suggests that the HASOE model overall captures Peruvian wealth, earnings, and consumption distributions reasonably well, even though only a small number of distributional moments are targeted.

Parameters  $\beta_R$  and  $\kappa_R$  appear only in the RASOE model. Parameter  $\beta_R$  is calibrated such that representative households’ Euler equation holds on the balanced growth path. Parameter  $\kappa_R$  is

<sup>25</sup>The real deposit interest rates are computed by deflating BCRP’s data on deposit rates in foreign currency (TIP-MEX) during 1992–2018 with the expected inflation on U.S. CPIs, which are constructed as in footnote 17.

<sup>26</sup>For this purpose, I use wealth inequality data from the World Inequality Database (WID). Since Peru does not have micro wealth data, the WID imputes the wealth inequality of Peru based on its income inequality and the wealth inequality of other countries that exhibit a similar degree of income inequality with Peru and have micro wealth data (Bajard, Chancel, Moshirif, and Piketty, 2022). As Alvarado, Atkinson, et al. (2021) note, the WID’s wealth concept is the market value of wealth, which includes financial assets yielding nonproductive pure rents, such as entrepreneurs’ claims to rents  $R_t^E$  in the model. Thus, I evaluate the market value of the claims assuming that entrepreneurs can trade the claims among themselves and include this value as part of entrepreneurs’ wealth. See Online Appendix F.

Table 2: Distributional Moments: Model (HASOE) vs. Data

Moment	Model	Data	Note	Data Source
<i>Wealth distribution</i>				
top 1.3% share	0.458	0.458	targeted	WID
top 5% share	0.532	0.644	untargeted	WID
top 10% share	0.594	0.760	untargeted	WID
Gini	0.642	0.874	untargeted	WID
<i>Earnings distribution of workers (residualized)</i>				
top 5% share	0.172	0.172	untargeted	ENAH0
top 10% share	0.279	0.273	untargeted	ENAH0
Gini	0.380	0.362	untargeted	ENAH0
<i>Earnings distribution of workers (unresidualized)</i>				
top 5% share	0.246	0.197	untargeted	ENAH0
top 10% share	0.373	0.311	untargeted	ENAH0
Gini	0.503	0.446	untargeted	ENAH0
<i>Consumption distribution of workers (residualized)</i>				
top 5% share	0.119	0.149	untargeted	ENAH0
top 10% share	0.211	0.238	untargeted	ENAH0
Gini	0.270	0.298	untargeted	ENAH0
<i>Consumption distribution of workers (unresidualized)</i>				
top 5% share	0.203	0.178	untargeted	ENAH0
top 10% share	0.320	0.283	untargeted	ENAH0
Gini	0.436	0.396	untargeted	ENAH0

Notes: ‘Residualized’ means that predictable components are removed, and ‘unresidualized’ means they are not.

calibrated such that the aggregate labor supply is normalized to 1 on the balanced growth path.

Table 3 reports the size of stock variables in the HASOE and RASOE models. The two models exhibit the same values of  $K/Y$  (10.91) and  $D/Y$  (2.487) as a result of calibration. The value of  $D/Y$  (2.487) is close to its data counterpart in Milesi-Ferretti and Lane (2017)’s dataset (1.783), although not directly targeted.<sup>27</sup> On the balanced growth path, the stock variables satisfy ‘ $K/Y =$

Table 3: The Size of Stock Variables

	$(K/Y)$	$(A/Y)$	$(A^W/Y)$	$(A^E/Y)$	$(B/Y)$	$(B^W/Y)$	$(D/Y)$
HASOE	10.91	7.785	6.016	142.2	0.633	0.642	2.487
RASOE	10.91	8.419	-	-	-	-	2.487

<sup>27</sup>See footnote 18 for a related discussion.

$A/Y + B/Y + D/Y$  in the HASOE model and  $K/Y = A/Y + D/Y$  in the RASOE model.<sup>28</sup> It is worth noting that  $B/Y$  (0.633) is very small in the HASOE model. This is because workers barely save in liquid assets in the model under the low deposit interest rate observed in data. Importantly, the model successfully captures the reality that households in emerging economies put most of their savings in nonfinancial, illiquid assets (Badarinza et al., 2019).

### 3.2 Bayesian Estimation

Parameters  $\psi, \phi, \theta_z, \theta_g, \rho_z, \sigma_z, \rho_g, \sigma_g, \rho_\mu$ , and  $\sigma_\mu$  govern the aggregate dynamics around the balanced growth path in both the RASOE and HASOE models. I estimate these parameters using standard Bayesian methods and macro data. For the macro data, I use output, consumption, investment, and trade-balance-to-output ratio, following the existing studies that conduct Bayesian estimation to examine emerging market business cycles. Specifically, I use the time series of  $[\Delta \log Y_t, \Delta \log C_t, \Delta \log I_t, \Delta(TB_t/Y_t)]$ , as in Chang and Fernández (2013).<sup>29</sup> I construct these data series using BCRP's national account data from 1980–2018.<sup>30</sup> In the estimation, I allow *i.i.d.* measurement errors on each data series and estimate their standard errors,  $\sigma_y^{me}$ ,  $\sigma_c^{me}$ ,  $\sigma_i^{me}$ , and  $\sigma_{tby}^{me}$ .

I construct a posterior distribution by sampling 300,000 draws through the Random Walk Metropolis Hastings (RWMH) algorithm described in Herbst and Schorfheide (2015) and burning the initial 50,000 draws. A successful implementation of the algorithm requires i) a variance-covariance matrix of the proposal distribution that is close to the variance-covariance matrix of the posterior distribution after scaling and ii) a scaling factor for the matrix that achieves an acceptance rate in the range 0.2–0.4. To this end, I run multiple preliminary stages of the RWMH algorithm and its variant before the main RWMH algorithm, through which i) the draws of the chain move close to the posterior mode, ii) the variance-covariance matrix of the proposal distribution is updated to become close to the variance-covariance matrix of the posterior distribution after scaling, and iii) the scaling factor is updated to achieve an acceptance rate close to 0.27.<sup>31</sup>

Table 4 presents the prior and posterior distributions. I impose a fairly flat prior distribution, as reported in the ‘Prior’ panel. Measurement errors are allowed to explain up to 6.25% of the

<sup>28</sup>In the HASOE model, the wealth-output ratio is 12.13, which is greater than the capital-output ratio because the market value of entrepreneurs’ claims to pure rents is included in wealth. See footnote 26 for a related discussion.

<sup>29</sup>Garcia-Cicco et al. (2010) use the same set of statistics except that they use  $TB_t/Y_t$  instead of  $\Delta(TB_t/Y_t)$ . Both choices are acceptable from a statistical perspective, as neither inherits a trend in the data and the model. I choose  $\Delta(TB_t/Y_t)$  over  $TB_t/Y_t$  because the countercyclicality of trade balance, a stylized pattern for both emerging and developed economies, is better captured when  $\Delta \log Y_t$  is correlated with  $\Delta(TB_t/Y_t)$  rather than with  $TB_t/Y_t$ .

<sup>30</sup>As described in footnote 17, I obtain BCRP’s real quarterly national account data, seasonally adjust them, and transform them to a per capita term. For the last step (transformation to a per capita term), I construct quarterly population series by linearly interpolating BCRP’s annual population data.

<sup>31</sup>As a result, I obtain acceptance rates of 0.278, 0.264, and 0.297 in the main RWMH algorithm for the estimation of the RASOE model in subsection 2.2, the HASOE model in subsection 2.1, and the HASOE model revised by augmenting a financial friction shock in section 4, respectively.

Table 4: Prior and Posterior Distributions of the Bayesian Estimation

	Density	Prior [Meta1,Meta2]	Posterior - RASOE ( $z, g, \mu$ )			Posterior - HASOE ( $z, g, \mu$ )		
			Mode	Mean	[0.05 ,0.95 ]	Mode	Mean	[0.05 ,0.95 ]
$\psi$	Uniform	[0.000,4.000]	0.513	2.471	[0.864,3.851]	0.005	0.004	[0.002,0.007]
$\phi$	Gamma	[15.00,15.00]	1.661	2.364	[1.095,3.976]	7.126	7.307	[5.823,8.970]
$\theta_z$	Uniform	[0.000,2.000]	0.017	0.245	[0.017,0.672]	0.000	0.001	[0.000,0.004]
$\theta_g$	Uniform	[0.000,2.000]	0.006	0.079	[0.003,0.255]	0.350	0.360	[0.270,0.456]
$\rho_z$	0.99·Beta	[0.500,0.200]	0.931	0.919	[0.873,0.954]	0.988	0.986	[0.979,0.989]
$\sigma_z$	Invgamma	[0.010,0.020]	0.017	0.017	[0.015,0.019]	0.029	0.030	[0.027,0.034]
$\rho_g$	0.99·Beta	[0.500,0.200]	0.988	0.988	[0.985,0.989]	0.044	0.065	[0.018,0.128]
$\sigma_g$	Invgamma	[0.010,0.020]	0.006	0.006	[0.006,0.007]	0.039	0.039	[0.034,0.045]
$\rho_\mu$	0.99·Beta	[0.500,0.200]	0.919	0.878	[0.746,0.961]	0.694	0.749	[0.609,0.868]
$\sigma_\mu$	Invgamma	[0.010,0.020]	0.011	0.047	[0.018,0.073]	0.003	0.003	[0.002,0.004]
$\rho_\eta$	0.99·Beta	[0.500,0.200]	-	-	-	-	-	-
$\sigma_\eta$	Invgamma	[0.010,0.020]	-	-	-	-	-	-
$\sigma_y^{me}$	Uniform	[0.000,0.007]	0.007	0.006	[0.006,0.007]	0.007	0.007	[0.006,0.007]
$\sigma_c^{me}$	Uniform	[0.000,0.009]	0.009	0.009	[0.008,0.009]	0.009	0.009	[0.008,0.009]
$\sigma_i^{me}$	Uniform	[0.000,0.045]	0.045	0.044	[0.043,0.045]	0.045	0.044	[0.043,0.045]
$\sigma_{tby}^{me}$	Uniform	[0.000,0.004]	0.004	0.004	[0.004,0.004]	0.004	0.004	[0.004,0.004]

Notes: Estimation is based on the Peruvian quarterly national account data from 1980–2018. In the prior density column, ‘0.99 · Beta’ means that the corresponding parameter multiplied by (1/0.99) follows a beta distribution. The column labeled ‘[Meta1,Meta2]’ reports the meta parameters of the prior distributions. For the uniform distribution, [Meta1,Meta2] is [lower bound, upper bound]. For the inverse gamma distribution and gamma distribution, [Meta1,Meta2] is [mean, standard deviation]. For ‘0.99 · Beta’, [Meta1,Meta2] is [mean, standard deviation] of the beta distribution part. Posterior statistics are based on 300,000 posterior draws from the RWMH algorithm, of which the initial 50,000 draws are burned.

observed variances. Parameters  $\rho_z$ ,  $\rho_g$ , and  $\rho_\mu$  in the prior distribution follow a beta distribution after being scaled by (1/0.99). This scaling is to ensure that these parameters do not exceed 0.99 under any posterior draw, as the precision of Auclert et al. (2021)’s computation method can be compromised when the economy becomes too persistent.<sup>32</sup> The ‘Posterior - RASOE ( $z, g, \mu$ )’ and ‘Posterior - HASOE ( $z, g, \mu$ )’ panels report key statistics of the posterior distributions for the RASOE and HASOE models introduced in subsections 2.2 and 2.1, respectively.

### 3.3 Model Performance

Table 5 reports each model’s prediction on key unconditional moments and log marginal likelihood after the Bayesian estimation. Starting from the data, Peruvian national accounts exhibit the stylized patterns of emerging market business cycles well. First, output is substantially more volatile than that of typical developed economies:  $\sigma(\Delta \log Y_t)$  is 0.027 in Peru, which far exceeds the average in rich economies, 0.008, reported in Uribe and Schmitt-Grohé (2017) (Table 1.6). Second, consumption is more volatile than output in Peru (excess consumption volatility):  $\sigma(\Delta \log C_t)$  is 0.036, which is substantially greater than  $\sigma(\Delta \log Y_t)$ , 0.027. Third, trade balance is countercyclical in Peru:  $\text{corr}(\Delta(TB_t/Y_t), \Delta \log Y_t)$  is -0.346. I highlight one more moment for later discussion, although it has received less attention in the literature;  $\text{corr}(\Delta \log C_t, \Delta \log I_t)$  is substantially less than one.<sup>33</sup> Moreover, the low correlation is not a particularity of Peru: the correlation is 0.189 for emerging countries and 0.278 for developed countries, on average.<sup>34</sup>

The RASOE model explains the data patterns reasonably well: consumption is more volatile than output ( $\sigma(\Delta \log C_t)/\sigma(\Delta \log Y_t) = 0.046/0.040 = 1.144$ ), trade balance is countercyclical ( $\text{corr}(\Delta(TB_t/Y_t), \Delta \log Y_t) = -0.111$ ), and the correlation between consumption and investment is low ( $\text{corr}(\Delta \log C_t, \Delta \log I_t) = 0.287$ ). The RASOE model also exhibits a few discrepancies with the data: output and consumption volatilities are noticeably greater than the data counterparts, and the output and consumption autocorrelations with two- and three-quarter lags are as substantial as those with a one-quarter lag. Overall, this model achieves a log marginal likelihood of 1108.61.

The HASOE model performs far more poorly than the RASOE model in terms of log marginal likelihood, yielding only 1061.50. This poor performance is also reflected in its prediction on un-

<sup>32</sup> Auclert et al. (2021)’s sequence space approach requires a truncation of sequences, and truncation errors can be nontrivial when the economy is extremely persistent. In this paper, I truncate sequences at  $T = 700$  when solving models and drop the last seven periods further when evaluating moments. As discussed in Online Appendix D, I verify that at the posterior mode, truncation errors are negligible in the model statistics used in this paper.

<sup>33</sup> It is indeed negative, but as we shall see later, what matters in this paper is that it is substantially less than one.

<sup>34</sup> In computing the correlation for emerging and developed countries, I use the quarterly macro data series and country categorization used for the business cycle statistics in Chapter 1 of Uribe and Schmitt-Grohé (2017). From the dataset, sample countries are selected if all five data series of output, investment, exports, imports, and consumption are available for at least twenty consecutive years. After the sample selection, 16 emerging countries and 17 rich countries remain in the sample. In averaging the correlation across multiple countries, I use population weights.



Table 5: Unconditional Moments and Marginal Likelihood

		$\Delta \log Y_t$	$\Delta \log C_t$	$\Delta \log I_t$	$\Delta(TB_t/Y_t)$
<i>Standard deviation</i>					
	RASOE ( $z, g, \mu$ ) model	0.040	0.046	0.143	0.029
	HASOE ( $z, g, \mu$ ) model	0.066	0.059	0.288	0.036
	HASOE ( $z, g, \mu, \eta$ ) model	0.029	0.036	0.167	0.018
	Data	0.027	0.036	0.179	0.017
<i>Contemporaneous correlation</i>					
<i>with <math>\Delta \log Y_t</math></i>	RASOE ( $z, g, \mu$ ) model		0.776	0.520	-0.111
	HASOE ( $z, g, \mu$ ) model		0.938	0.913	-0.824
	HASOE ( $z, g, \mu, \eta$ ) model		0.598	0.509	-0.248
	Data		0.681	0.437	-0.346
<i>with <math>\Delta(TB_t/Y_t)</math></i>	RASOE ( $z, g, \mu$ ) model		-0.461	-0.610	
	HASOE ( $z, g, \mu$ ) model		-0.821	-0.921	
	HASOE ( $z, g, \mu, \eta$ ) model		-0.180	-0.632	
	Data		-0.318	-0.460	
<i>with <math>\Delta \log C_t</math></i>	RASOE ( $z, g, \mu$ ) model			0.287	
	HASOE ( $z, g, \mu$ ) model			0.788	
	HASOE ( $z, g, \mu, \eta$ ) model			-0.223	
	Data			-0.158	
<i>Autocorrelation</i>					
<i>with lag 1</i>	RASOE ( $z, g, \mu$ ) model	0.474	0.312	-0.202	-0.229
	HASOE ( $z, g, \mu$ ) model	-0.165	-0.028	-0.280	-0.126
	HASOE ( $z, g, \mu, \eta$ ) model	0.036	-0.048	-0.080	-0.131
	Data	0.404	0.078	-0.304	0.023
<i>with lag 2</i>	RASOE ( $z, g, \mu$ ) model	0.472	0.312	-0.046	-0.079
	HASOE ( $z, g, \mu$ ) model	0.006	0.008	-0.032	-0.044
	HASOE ( $z, g, \mu, \eta$ ) model	0.027	-0.039	-0.067	-0.078
	Data	0.009	0.036	-0.094	-0.077
<i>with lag 3</i>	RASOE ( $z, g, \mu$ ) model	0.471	0.312	0.006	-0.028
	HASOE ( $z, g, \mu$ ) model	0.015	0.001	-0.015	-0.041
	HASOE ( $z, g, \mu, \eta$ ) model	0.022	-0.025	-0.053	-0.058
	Data	-0.090	-0.112	0.026	-0.061
<i>Log marginal likelihood</i>					
	RASOE ( $z, g, \mu$ ) model			1108.61	
	HASOE ( $z, g, \mu$ ) model			1061.50	
	HASOE ( $z, g, \mu, \eta$ ) model			1155.54	

Notes: Unconditional moments are computed under each posterior draw, and the means across the posterior distribution are reported. Log marginal likelihood is computed according to Geweke (1999)'s Modified Harmonic Mean method under truncation parameter 0.1. Each model's log marginal likelihood barely changes over different values of the truncation parameter.

conditional moments, which fails to explain data patterns in several important dimensions: output and consumption are far greater than the data counterparts, consumption is less volatile than output ( $\sigma(\Delta \log C_t)/\sigma(\Delta \log Y_t) = 0.059/0.066 = 0.897$ ), trade balance is excessively countercyclical ( $\text{corr}(\Delta(TB_t/Y_t), \Delta \log Y_t) = -0.824$ ), and the correlation between consumption and investment is close to 1 ( $\text{corr}(\Delta \log C_t, \Delta \log I_t) = 0.788$ ).

How does the RASOE model explain the data patterns? By contrast, why does the HASOE model fail to do so? The following subsection addresses these questions.

### 3.4 How RASOE Works and Why HASOE Doesn't

I start by examining the driving mechanism of the RASOE model. The ‘RASOE ( $z, g, \mu$ ) model’ panel in Table 6 presents variance decomposition in the RASOE model and shows that trend shocks are the main driver of output and consumption fluctuations: 50.8% of output fluctuations and 83.6% of consumption fluctuations are driven by trend shocks. This result is consistent with [Aguiar and Gopinath \(2007\)](#)’s finding that trend shocks drive emerging market business cycles.

In Figure 2a, I examine the consumption response to a trend shock in the RASOE model and verify that the permanent income effect of a trend shock drives excess consumption volatility, exactly as [Aguiar and Gopinath \(2007\)](#) argue. The first panel shows that the impact effect of a trend shock on consumption is greater than that on output. The RASOE model imposes the GHH preference, under which households smooth per-period utility  $GHH_t := C_t - h_t(L_t)$  subject to budget constraints, where  $h_t(L_t) := \kappa_R X_{t-1} L_t^{1+\omega}$  is labor disutility. Since  $C_t = GHH_t + h_t(L_t)$ , the strong impact effect of a trend shock on consumption can come either from  $GHH_t$  or from  $h_t(L_t)$ . In the second panel, I decompose the consumption response into the responses of  $GHH_t$  and  $h_t(L_t)$  and find that almost all the impact effect on consumption comes from the impact effect on  $GHH_t$ . A trend shock affects  $GHH_t$  by affecting  $w_t$  and  $r_t^a$  in the budget constraints and labor disutility function  $h_t(\cdot)$  (via  $X_{t-1}$  in it). In particular, the effects on  $w_t$  and  $h_t(\cdot)$  transmit to  $GHH_t$  through earnings  $w_t L_t$  in the budget constraints, as labor supply  $L_t$  is determined by wage  $w_t$  and labor disutility  $h_t(\cdot)$ . In the third panel, I decompose the response of  $GHH_t$  into the response driven by  $w_t$  and  $h_t(\cdot)$  and the response driven by  $r_t^a$  and find that it is dominantly driven by the former.<sup>35</sup> The last panel plots the impulse response of earnings  $w_t L_t$ , which mildly jumps on impact but grows strongly in the future.<sup>36</sup> These panels show that the permanent income effect of the future growth of earnings generates a strong impact effect on  $GHH_t$  and thus on  $C_t$ .<sup>37</sup>

In the literature, a competing hypothesis exists on the driving mechanism of emerging market

<sup>35</sup>The consumption response decomposition into driving factors in the RASOE model requires an extra computational step in addition to solving the model (unlike that in the HASOE model). See Online Appendix G for details.

<sup>36</sup>In fact, the impulse response of  $w_t L_t$  is the same as that of  $Y_t$  because  $w_t L_t = (1 - \alpha)Y_t$  in equilibrium.

<sup>37</sup>For impulse responses of  $I$  and  $TB/Y$  in the RASOE ( $z, g, \mu$ ) model, see Figure M.2a in Online Appendix M.

Table 6: Variance Decomposition

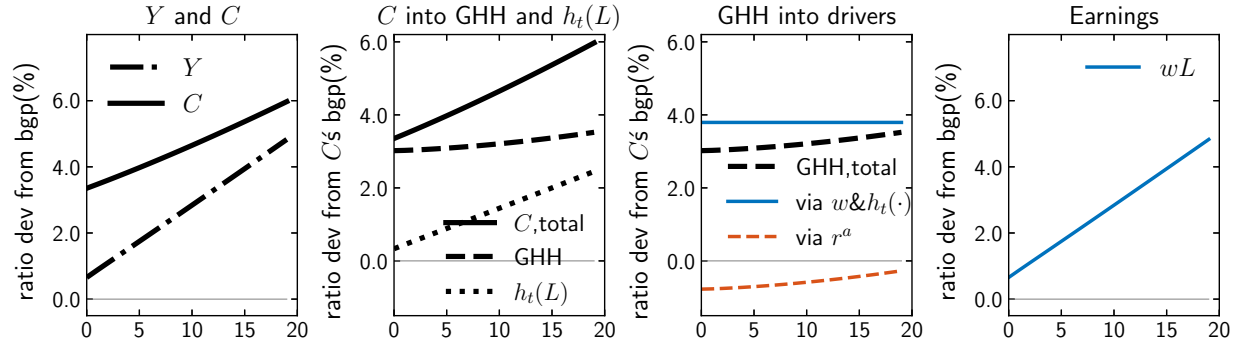
	$\Delta \log Y_t$	$\Delta \log C_t$	$\Delta \log I_t$	$\Delta(TB_t/Y_t)$
<i>RASOE (z, g, <math>\mu</math>) model</i>				
stationary productivity shock (z)	0.492 (0.048)	0.163 (0.026)	0.341 (0.053)	0.020 (0.015)
trend shock (g)	0.508 (0.048)	0.836 (0.026)	0.056 (0.013)	0.353 (0.041)
interest rate shock ( $\mu$ )	0.001 (0.000)	0.001 (0.001)	0.603 (0.054)	0.627 (0.042)
<i>HASOE (z, g, <math>\mu</math>) model</i>				
stationary productivity shock (z)	0.547 (0.043)	0.826 (0.030)	0.236 (0.033)	0.370 (0.045)
trend shock (g)	0.452 (0.043)	0.164 (0.030)	0.708 (0.036)	0.380 (0.046)
interest rate shock ( $\mu$ )	0.000 (0.000)	0.010 (0.002)	0.056 (0.011)	0.251 (0.038)
<i>HASOE (z, g, <math>\mu</math>, <math>\eta</math>) model</i>				
stationary productivity shock (z)	0.918 (0.024)	0.318 (0.052)	0.189 (0.039)	0.021 (0.011)
trend shock (g)	0.078 (0.023)	0.030 (0.013)	0.379 (0.048)	0.929 (0.044)
interest rate shock ( $\mu$ )	0.000 (0.000)	0.000 (0.000)	0.009 (0.012)	0.034 (0.042)
financial friction shock ( $\eta$ )	0.004 (0.002)	0.652 (0.051)	0.423 (0.043)	0.016 (0.015)

*Notes:* The decomposed shares are computed under each posterior draw, and their means and standard deviations across the posterior distribution are reported. The numbers in parentheses are the posterior standard deviations.

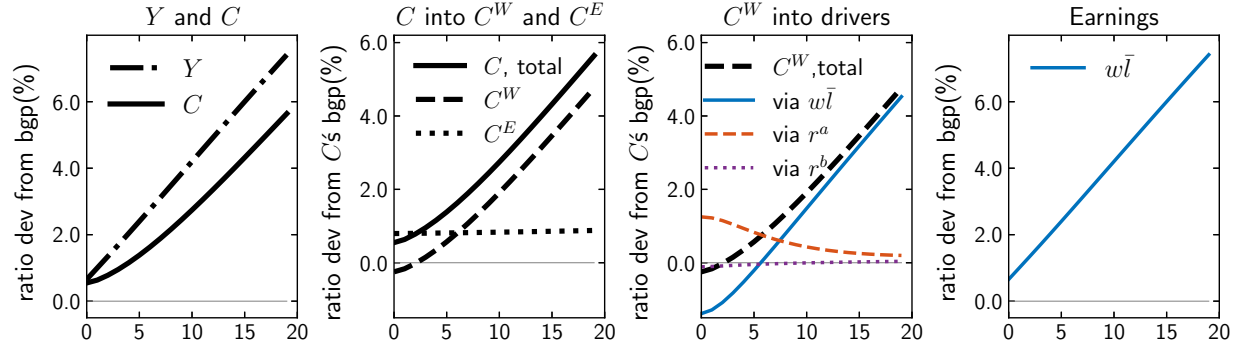
business cycles: emerging economies face volatile interest rates, and they cause excess consumption volatility because households respond to them by intertemporally substituting consumption (Neumeyer and Perri, 2005). This mechanism is muted in explaining the Peruvian data largely because of the low correlation between consumption and investment; when the interest rate increases, both consumption and investment plunge<sup>38</sup>; thus, if interest rates drove business cycles, consumption and investment would be strongly positively correlated.<sup>39</sup>

<sup>38</sup>Investment decreases because the marginal rate of capital should increase toward the interest rate after reflecting capital adjustment costs. See equation (A.14) in Online Appendix A.

<sup>39</sup>Rather, low  $\text{corr}(\Delta \log C_t, \Delta \log I_t)$  is achieved in the RASOE model by appointing two different shocks,  $g$  and  $\mu$ , as the main drivers of consumption and investment fluctuations, respectively. Moreover, a highly persistent  $\rho_g$  helps



(a) RASOE Model Prediction under the RASOE Posterior



(b) HASOE Model Prediction under the RASOE Posterior

Figure 2: Consumption Response to  $g$  Shock: RASOE vs. HASOE under the RASOE Posterior

Notes: Figures 2a and 2b plot the impulse response of consumption and other related variables to a one-standard-deviation trend shock in the RASOE and HASOE models, respectively, evaluated at the same parameter draws from the RASOE model's posterior distribution. The model statistics are computed at each posterior draw, and their means across the posterior distribution are plotted. The unit of the y-axis is either 'ratio dev from bgp(%)', which represents the deviation from the balanced growth path (b.g.p) of the variable of interest divided by its value on the b.g.p, or 'ratio dev from  $C$ 's bgp(%)', which represents the deviation of the variable of interest divided by the value of  $C_t$  on the b.g.p.

Now, I turn to the failure of the HASOE model. The 'HASOE ( $z, g, \mu$ ) model' panel in Table 6 presents variance decomposition in the HASOE model and shows that trend shocks do not play an important role in generating consumption fluctuations. Instead, stationary productivity shocks ( $z$ ) generate most consumption fluctuations (82.6%). As we see in Table 5, however, stationary productivity shocks cannot generate excess consumption volatility.

Why can't the Aguiar and Gopinath (2007) mechanism operate in the HASOE model as it does in the RASOE model? To answer this question, I feed the RASOE model's posterior parameter draws into the HASOE model and examine how consumption responds to a trend shock. The first

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the RASOE model generate a low correlation between consumption and investment growth, as the shock generates a slightly negative correlation: in response to a trend shock, consumption jumps far more than output, leaving no room for investment growth (even with capital inflow) and rather inducing a decrease in investment.

panel in Figure 2b plots consumption and output responses in this experiment. The impact effect on output is similar between the RASOE and HASOE models, and output grows more strongly in the HASOE model. Despite this stronger future output growth (and a consequent stronger permanent income effect), the impact effect on consumption in the HASOE model is substantially weaker than that in the RASOE model and is only as much as the impact effect on output. Since  $C_t = pC_t^W + (1-p)C_t^E$  in the HASOE model, the muted initial response of consumption might come from either  $C_t^W$  or  $C_t^E$ . In the second panel, I decompose the consumption response into the responses of  $C_t^W$  and  $C_t^E$  and find that the muted impact effect on consumption comes entirely from the muted initial response of  $C_t^W$ . A trend shock affects workers' consumption by affecting  $w_t\bar{l}_t$ ,  $r_t^a$ , and  $r_t^b$  in their budget constraints. In the third panel, I decompose the response of  $C_t^W$  into the responses driven by  $w_t\bar{l}_t$ ,  $r_t^a$ , and  $r_t^b$  and find that the muted initial response of  $C^W$  is entirely driven by its negative (!) initial response to aggregate earnings,  $w\bar{l}$ .<sup>40</sup> The last panel plots the impulse response of  $w\bar{l}$ , which mildly jumps on impact, as in the RASOE model, but grows more rapidly in the following periods than it does in the RASOE model.

At first, it may seem surprising that workers do not increase and rather decrease consumption in response to a strong future growth of aggregate earnings and a consequent permanent income increase. Behind this result, two economic forces work against the permanent income effect. First, workers face large financial friction, which makes it difficult for them to borrow from the future by cashing out assets. As a result, despite a large permanent income increase, workers cannot increase their consumption accordingly (financial friction effect). Second, because workers' earnings are determined by aggregate earnings  $w_t\bar{l}_t$  multiplied by idiosyncratic productivity  $\Gamma_i e_{i,t}$ , where  $e_{i,t}$  bears idiosyncratic earnings risk, the future growth of aggregate earnings means that workers must face a greater idiosyncratic earnings risk in the future. The greater future idiosyncratic risk enhances workers' precautionary saving, further depressing a consumption response to a trend shift (precautionary saving effect).<sup>41</sup> In short, large financial friction and correspondingly strong precautionary saving hinder the Aguiar and Gopinath (2007) mechanism in the HASOE model.<sup>42</sup>

<sup>40</sup>To be precise, aggregate earnings of the economy are  $w_t L_t = (p\bar{\Gamma}\bar{e})w_t\bar{l}_t$ , and  $w_t\bar{l}_t$  should be named an 'aggregate earnings per efficiency unit.' Given that  $w_t L_t$  is a scaled-up version of  $w_t\bar{l}_t$  and they exhibit the same impulse responses, I refer to both terms as 'aggregate earnings' for brevity unless a distinction between the two is necessary.

<sup>41</sup>The effect of enhanced precautionary saving in response to a permanent income change is also studied by Carroll (2009) but in an environment without financial friction. The author finds only a modest effect (a 8%–15% weaker consumption response than the hypothetical one-for-one response to a permanent income change). The effect is stronger in my model because financial friction amplifies the precautionary saving motive. Regardless of this difference, Carroll (2009) provides an insightful explanation about the enhanced precautionary saving effect through the lens of his prominent buffer-stock model: in his model, households have a target wealth-to-permanent-income ratio, *i.e.*, households save when the ratio is below the target and dissave when above the target; given the initial level of wealth, a positive permanent income shock decreases the ratio, inducing households to save more, and this enhanced precautionary saving weakens a consumption response to the shock.

<sup>42</sup>Decomposing these two forces is difficult because financial friction and precautionary saving affect each other: financial friction makes workers more concerned about a future low income path, strengthening their precautionary

## 4 What is Missing: Volatile Domestic Financial Condition

Thus far, I have shown the following: the RASOE model can explain macro-level stylized patterns well, while it fails to match micro-level consumption behavior; on the other hand, my HASOE model can successfully capture the micro-level consumption behavior, while it fails to explain the macro-level stylized patterns.

In the face of this dilemma, I explore whether there is any important economic condition that exists in reality but is omitted in my HASOE model. And I indeed find one: Peruvian households face very high and volatile finance rates when they have to borrow for consumption. Figure 3 plots the real finance rate on consumer loans in Peru (blue solid line) and compares it with i) the Peruvian real interest rate in the international financial market, which corresponds to  $r_t$  in the model (purple dotted line), and ii) the real finance rate on U.S. consumer loans (red dashed line).<sup>43</sup> This figure shows that when they have to borrow, Peruvian households face substantially more high and volatile finance rates than  $r_t$  in the model and compared to U.S. households.

How can we interpret this volatile domestic financial condition in the model? As discussed

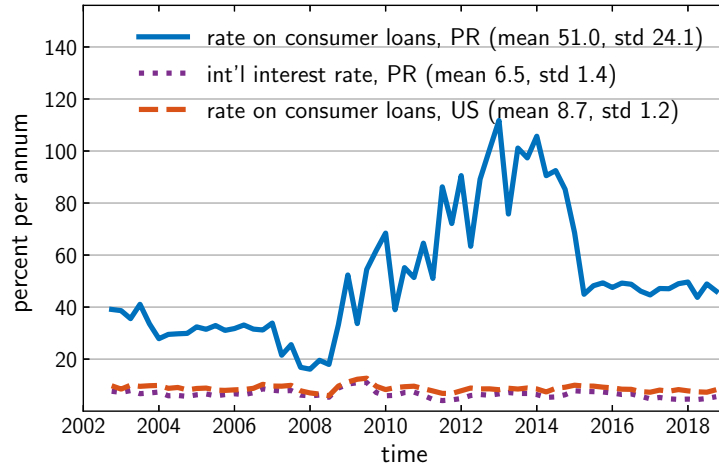


Figure 3: Real Finance Rates on Consumer Loans in Peru

*Notes:* This figure plots the real finance rate on consumer loans in Peru (blue solid line) and compares it with the real interest rate of Peru in the international financial market (purple dotted line) and the real finance rate on U.S. consumer loans (red dashed line). Data Sources: SBS, BCRP, FRB.

saving; strong precautionary saving behavior makes workers save more against financial friction and pay more adjustment costs. Instead, in Online Appendix H, I scale down either financial friction ( $\chi_1$ ) or idiosyncratic risk ( $\sigma_{e_1}$ ,  $\sigma_{e_2}$ ) by half and see how workers' consumption response to  $w\bar{l}$  changes. Under reduced idiosyncratic risk, the initial response of workers' consumption turns positive, reaching approximately 1% of its balanced growth path. Under reduced financial friction, the initial response is not only positive but also quite large, reaching above 2%.

<sup>43</sup>Online Appendix J provides details of the data construction for the finance rate series on consumer loans in Peru and the other two compared series in Figure 3.



above, households in emerging economies put most of their savings in nonfinancial, illiquid assets in reality (Badarinza et al., 2019), and my HASOE model captures this pattern well (see Table 3). In reality, these assets are illiquid because they are difficult to trade frequently, but in the model, illiquidity is captured such that households can trade these assets each period with an adjustment cost. Moreover, the illiquid asset  $a_{i,t}$  in the model represents a net asset position (*i.e.*, gross assets - debt). Under such model specification, a realistic interpretation of an illiquid asset adjustment cost is a ‘haircut out of collateralized debt’: households collateralize  $|v_{i,t}|$  amount of illiquid assets and cash out  $|v_{i,t}| - \chi_t(v_{i,t})$ , with haircut  $\chi_t(v_{i,t})$ , from a bank or a pawnshop.<sup>44</sup>

In Online Appendix I, I formalize this haircut interpretation by providing a microfoundation, closely following Fostel and Geanakoplos (2015), in which the illiquid asset adjustment cost is the haircut of collateralized borrowing. This microfoundation provides two important takeaways for the model. First, both the borrowers’ default risk and the market liquidity risk are important determinants of the haircut size (or, equivalently, the illiquid asset adjustment cost in the model). Second, the part of the haircut rate due to borrowers’ default risk can be recovered from the spread between the finance rate of the borrowing and the asset return rate, which corresponds to the gap between the blue solid line and the purple dotted line in Figure 3.<sup>45,46</sup>

This microfoundation sheds light on how to interpret the volatile domestic financial condition through the lens of the HASOE model: the large and volatile gap between the blue solid line and purple dotted line in Figure 3 means substantial haircut fluctuations due to the borrowers’ default risk. Under this interpretation, I incorporate the volatile domestic financial condition of Peru by augmenting a financial friction shock to the illiquid asset adjustment cost. Specifically, working households’ budget constraint (2) is revised as follows.

$$c_{i,t} + b_{i,t} + v_{i,t} + \eta_t \chi_t(v_{i,t}, a_{i,t-1}; \Gamma_i) = w_t \Gamma_i e_{i,t} \bar{l}_t + (1 - \xi)(1 + r_t^b) b_{i,t-1}, \quad (21)$$

<sup>44</sup>The explanation of the haircut interpretation above assumes that workers cash out illiquid assets, *i.e.*,  $v_{i,t} < 0$ . This is indeed the case for most workers (87.6%) on the balanced growth path. For the rest of the workers (12.4%) with  $v_{i,t} > 0$ , the haircut interpretation still fits under the following assumption: when workers increase the net illiquid asset position by the amount of  $|v_{i,t}|$ , they have to purchase  $(2 \times |v_{i,t}|)$  in illiquid assets and collateralize  $(|v_{i,t}|)$  portion of the assets with haircut  $\chi_t(v_{i,t})$  due to, for example, the granularity of illiquid assets.

<sup>45</sup>Specifically, the part of the haircut rate due to borrowers’ default risk equals  $(r_t^{C,J} - r_t^J)/(1 + r_t^{C,J})$ , where  $r_t^{C,J}$  is the finance rate of the borrowing with  $J$ -period maturity and  $r_t^J$  is the expected return rate on the collateralized illiquid assets over the  $J$  periods. The gap between the blue solid line and the purple dotted line in Figure 3 corresponds to the numerator of the fraction under  $J = 4$ . See equation (I.5) in Online Appendix I.

<sup>46</sup>On the balanced growth path of the model, most workers (87.6%) cash out their illiquid assets (*i.e.*,  $v_{i,t} < 0$ ), and importantly, they do so for immediate consumption smoothing rather than for running a personal business or purchasing large assets. In this regard, the collateralized borrowing involved in this asset liquidation within the model is best represented by consumer loans among the available credit categories in Peruvian data (firm loans, consumer loans, mortgages, and credit cards). In the data, consumer loans exclude credit card debt, which is typically not backed by collateral.

where  $\eta_t$  is the financial friction shock. The aggregation equation (10) is revised accordingly:

$$\chi_t^{agg} = p\chi_t^W, \quad \chi_t^W = \int_{\Gamma} \int_{e_1, e_2, b_-, a_-} \eta_t \chi_t(a_t(e_1, e_2, b_-, a_-; \Gamma) - (1 + r_t^a)a_-, a_-; \Gamma) d\Psi_t dG. \quad (22)$$

The balanced-growth-path value of  $\eta_t$  is 1, and  $\log \eta_t$  follows an AR(1) process:

$$\log \eta_t = \rho_{\eta} \log \eta_{t-1} + \varepsilon_t^{\eta}, \quad \varepsilon_t^{\eta} \sim N(0, \sigma_{\eta}^2). \quad (23)$$

After adding the financial friction shock, I re-estimate the HASOE model.<sup>47</sup> I impose the same prior distribution as before on all the preexisting parameters. For newly added parameters  $\rho_{\eta}$  and  $\sigma_{\eta}$ , I impose the same prior distribution as the one imposed on the other exogenous shock processes. See the ‘Prior’ panel in Table 4 for details. The ‘Posterior - HASOE ( $z, g, \mu, \eta$ )’ panel in Table 4 reports key statistics of the posterior distribution of the revised HASOE model.

Table 5 reports the business cycle moments predicted by the revised HASOE model and its marginal likelihood in the rows labeled ‘HASOE ( $z, g, \mu, \eta$ ) model.’ The revised HASOE model achieves a log marginal likelihood of 1155.54, which is substantially higher than that of the previous HASOE model as well as the RASOE model. This success is reflected in its prediction on unconditional moments, which explains data patterns quite well: consumption is more volatile than output ( $\sigma(\Delta \log C_t)/\sigma(\Delta \log Y_t) = 0.036/0.029 = 1.215$ ), trade balance is countercyclical to a correct degree ( $\text{corr}(\Delta(TB_t/Y_t), \Delta \log Y_t) = -0.248$ ), and the correlation between consumption and investment is low ( $\text{corr}(\Delta \log C_t, \Delta \log I_t) = -0.223$ ). The revised HASOE model also predicts output and consumption volatilities close to their data counterparts ( $\sigma(\Delta \log Y_t) = 0.029$  in

<sup>47</sup> A common misconception about Bayesian estimation is that perfect moment matching should be achieved when the number of shocks equals that of observed variables. This misconception might originate from a fact about a related but different exercise: when the number of shocks equals that of observed variables, one can recover shocks using a smoothing technique such that the model fully replicates the observed time series (data). The intuition behind this fact is simple:  $n_{obs}T_{obs}$  unknowns (shocks) are used to match  $n_{obs}T_{obs}$  targets (data), where  $n_{obs}$  is the number of observed variables and  $T_{obs}$  is the length of the observed time series. (See Online Appendix L for an actual smoothing exercise.) Bayesian estimation, on the other hand, aims to simulate a distribution of posterior log-likelihood, which is the sum of log prior density and log likelihood. Under the normality assumptions on aggregate shocks, the log-likelihood equals  $-\frac{1}{2} \log(|\Sigma|) - \frac{1}{2} \mathbf{y}' \Sigma^{-1} \mathbf{y}$  (ignoring constant terms), where  $\mathbf{y}$  is an  $(n_{obs}T_{obs} \times 1)$ -vector of a stacked time series of demeaned observed variables (data), and  $\Sigma$  is the model-predicted variance-covariance matrix of  $\mathbf{y}$ . If one can freely choose  $\Sigma$  (or, equivalently, its  $\frac{1}{2}n_{obs}T_{obs}(n_{obs}T_{obs} + 1)$  elements, given the symmetry requirement), the log-likelihood is maximized at the empirical variance-covariance matrix  $\hat{\Sigma} := \mathbf{y}\mathbf{y}'$ . In the Bayesian estimation, however,  $\Sigma$  is a model object represented by  $n_{obs}T_{obs} + \frac{1}{2}n_{obs}(n_{obs} - 1)(2T_{obs} - 1)$  business cycle moments, including variances and autocovariances ( $n_{obs}T_{obs}$ ) as well as cross-variable covariances and autocovariances ( $\frac{1}{2}n_{obs}(n_{obs} - 1)(2T_{obs} - 1)$ ). Ultimately, these moments are determined by an  $(n_{\Theta} \times 1)$ -vector of Bayesian-estimated parameters  $\Theta$ . In short,  $n_{\Theta}$  unknowns (the Bayesian-estimated parameters) are used to represent  $n_{obs}T_{obs} + \frac{1}{2}n_{obs}(n_{obs} - 1)(2T_{obs} - 1)$  business cycle moments appearing (most of them repeatedly) in  $\Sigma$  to maximize posterior likelihood. Thus, as long as  $n_{\Theta} < n_{obs}T_{obs} + \frac{1}{2}n_{obs}(n_{obs} - 1)(2T_{obs} - 1)$ , perfect moment matching is not achieved in this exercise. In the Bayesian estimation of the HASOE ( $z, g, \mu, \eta$ ) model conducted above,  $n_{\Theta} = 16$  and  $n_{obs}T_{obs} + \frac{1}{2}n_{obs}(n_{obs} - 1)(2T_{obs} - 1) = 2,474$ .

the model, 0.027 in data;  $\sigma(\Delta \log C_t) = 0.036$  in the model, 0.036 in data). Other business cycle moments reported in Table 5 are also closely predicted by this model.<sup>48</sup>

The ‘HASOE ( $z, g, \mu, \eta$ ) model’ panel in Table 6 presents variance decomposition in the revised HASOE model. Most output fluctuations (91.8%) are driven by stationary productivity shocks, while trend shocks play only a limited role (7.8%). Consumption fluctuations are almost entirely driven by financial friction shocks (65.2%) and stationary productivity shocks (31.8%), while trend shocks play essentially no role.<sup>49</sup>

The addition of the financial friction shock is effective at reviving the HASOE model for two reasons. First, the  $\eta$  shock generates large consumption fluctuations while causing small output fluctuations (see Table 6), resolving the absence of excess consumption volatility in the initial HASOE model. Second, the  $\eta$  shock also generates large investment fluctuations (see Table 6), and importantly, the consumption and investment responses are in the opposite direction (see Figure M.2b in Online Appendix M), generating a negative correlation between consumption and investment and thus fixing the strongly positive  $\text{corr}(\Delta \log C_t, \Delta \log I_t)$  in the initial HASOE model.<sup>50,51</sup>

To identify the driving mechanism of consumption fluctuations, I examine consumption re-

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<sup>48</sup>One exception to this successful outcome is the autocorrelation of  $\Delta \log Y_t$  with a one-quarter lag (0.036 in the model, 0.404 in data). However, this discrepancy quickly dissipates from a two-quarter lag forward. Given that the discrepancy survives only one quarter and that output fluctuations mostly come from stationary productivity shocks (see the ‘HASOE( $z, g, \mu, \eta$ ) model’ panel in Table 6), it is likely that replacing the conventional AR(1) process of stationary productivity shocks with an ARMA(1,1) process can fix this discrepancy. However, I do not impose this rather unconventional assumption, as the model aims to minimize changes from conventional representative-agent models other than the heterogeneous household block with high MPCs.

<sup>49</sup>The ‘HASOE ( $z, g, \mu, \eta$ ) model’ panel in Table 6 also reports the variance decomposition of investment and trade-balance-to-output ratio fluctuations in the revised HASOE model. Notably, a trend shock ( $g$ ) plays an important role in these fluctuations. When firms expect future productivity growth in the model, they can increase investment without a precautionary saving concern, unlike workers’ consumption decisions. Moreover, given the limited output response to a  $g$  shock, firms finance investment by rapidly increasing international debt through banks. However, a  $g$  shock is dispensable in these roles. In Online Appendix K, I show that the revised HASOE model can also explain the macro data well even without a  $g$  shock. In the absence of a  $g$  shock, an interest rate shock ( $\mu$ ) accounts for most of the investment and trade-balance-to-output ratio fluctuations that used to be generated by a  $g$  shock.

<sup>50</sup>Consumption and investment respond to an  $\eta$  shock in the opposite direction for the following reason. When financial friction is heightened, workers reduce consumption and increase saving due to either consumption smoothing disruption or enhanced precautionary saving, as will be discussed further in a later part of this section. As aggregate savings increase, the interest rate decreases, and the lower interest rate boosts investment.

<sup>51</sup>There are other shocks studied in the literature that can also exhibit the two features of the  $\eta$  shock described above, such as a preference shock on household utility and a Justiniano, Primiceri, and Tambalotti (2010)-type investment shock. In Online Appendix K, I show that each of these shocks can also revive the HASOE model, achieving an even slightly higher marginal likelihood than the  $\eta$  shock. However, I prefer the financial friction shock to these alternative shocks for the following reasons. First, the financial friction shock has a clear empirical motivation (Figure 3) other than business cycle moments. Second, a preference shock is difficult to measure and vague to interpret (as it can be mapped into too many things in reality). Third, an investment shock fixes the HASOE model by creating negative correlation between consumption and investment through an economically implausible mechanism: household consumption sharply drops when an investment technology becomes more efficient. (Justiniano et al. (2010) do not have this problem by allowing the capital utilization rate to increase when the investment technology becomes more efficient. Then, the shock cannot fix the HASOE model because it will no longer generate the negative correlation.)

sponses to a financial friction shock and a stationary productivity shock. Since  $C_t = pC_t^W + (1 - p)C_t^E$  in the HASOE model, the total consumption response can be decomposed into the responses of  $C_t^E$  and  $C_t^W$ . Moreover, since aggregate shocks affect workers' consumption by affecting  $w_t\bar{l}_t$ ,  $r_t^a$ ,  $r_t^b$ , and  $\eta_t$  in their budget constraints, the response of  $C_t^W$  can be further decomposed into the responses driven by each of them.

Figure 4a plots the total consumption response to a financial friction shock and decomposes it into entrepreneurs' response and workers' responses driven by  $w\bar{l}$ ,  $r^a$ ,  $r^b$ , and  $\eta$  in their budget constraints. This figure shows that the total consumption response almost entirely comes from workers' consumption response to the change in  $\eta_t$  in their budget constraints. On the balanced growth path, most workers (87.6%) cash out their illiquid assets (*i.e.*,  $v_{i,t} < 0$ ) to smooth consumption to the extent they can. When financial friction is heightened, they fail to smooth consumption more significantly, as it becomes more costly to cash out their assets. Moreover, heightened financial friction makes households more concerned about a future low income path, enhancing their precautionary saving and thus further depressing their consumption.

Figure 4b plots the total consumption response to a stationary productivity shock and its decomposed responses. This figure shows that the total consumption response mostly comes from workers' consumption response driven by aggregate earnings  $w_t\bar{l}_t$ . Workers' consumption response driven by  $r_t^a$  also nontrivially contributes to the total response. The economic mechanism behind this result is as follows. When a positive stationary productivity shock hits the economy, both la-

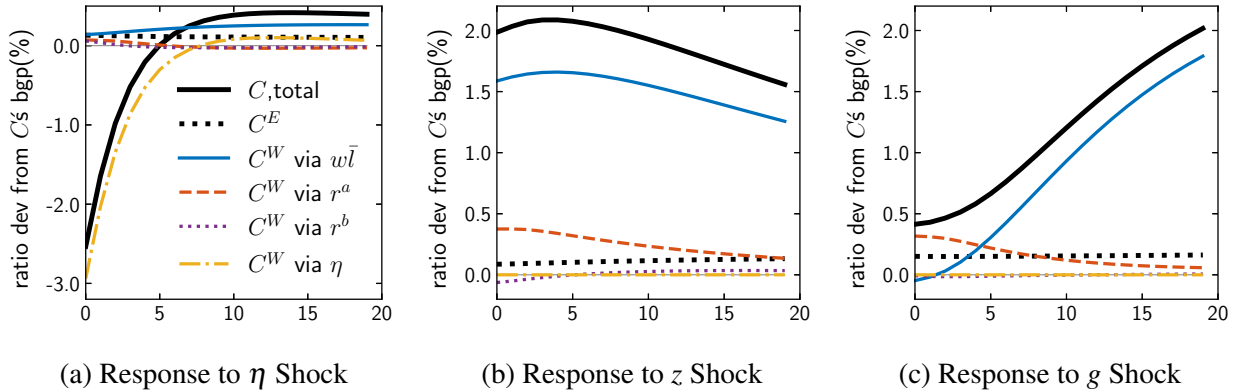


Figure 4: Consumption Response Decomposition in the HASOE ( $z, g, \mu, \eta$ ) Model

*Notes:* Figures 4a, 4b, and 4c plot the consumption response to one-standard-deviation  $\eta$ ,  $z$ , and  $g$  shocks, respectively, in the revised HASOE model. In each figure, the consumption response is decomposed into entrepreneurs' response and workers' responses driven by  $w\bar{l}$ ,  $r^a$ ,  $r^b$ , and  $\eta$  in their budget constraints. The model statistics are computed at each posterior draw, and their means across the posterior distribution are plotted. The unit of the y-axis is 'ratio dev from  $C$ 's bgp(%)', which represents the deviation from the balanced growth path (b.g.p) of the variable of interest divided by the value of  $C_t$  on the b.g.p.

bor and investment demands increase, and thus, aggregate earnings ( $w_t \bar{l}_t$ ,  $t \geq 0$ ) and interest rates ( $r_t = r_{t+1}^a$ ,  $t \geq 0$ ) increase. Moreover, the asset price ( $q_0$ ) jumps on impact (due to the higher future productivity), and so does the rate of asset return ( $r_0^a$ ). As a result, workers' income, including their earnings and asset returns, increases. Importantly, because workers exhibit high MPC, they strongly translate these income fluctuations into consumption fluctuations.

In addition to the consumption responses to the main shocks ( $z$  and  $\eta$ ), I also examine the consumption response to a trend shock. Figure 4c plots the total consumption response to a trend shock and its decomposed responses and reconfirms the economic intuition obtained in the previous section: the consumption response to a trend shock is muted in the HASOE model because of financial friction and enhanced precautionary saving.<sup>52</sup>

Although this paper focuses on aggregate-level business cycles, the model is also suitable for examining distributional consequences of aggregate shocks. In Figure M.3 of Online Appendix M, I plot workers' consumption ( $C^W$ ) response to each aggregate shock within the whole group of workers and within the bottom and top residual earnings ( $e_{i,t}$ ) deciles. I find that on impact, a financial friction shock hits the bottom decile much more strongly, a stationary productivity shock hits the bottom and top deciles almost equally, and a trend shock hits the top decile more strongly.<sup>53</sup>

## 5 Counterfactual Experiment

As discussed in the previous section, Peruvian households' high MPC and correspondingly strong financial friction and precautionary saving play key roles in the transmission mechanism of  $\eta$  and  $z$  shocks to generate consumption volatility in the HASOE model. In this section, I quantify their role by running a counterfactual experiment under which household MPC is adjusted to the U.S. level, which is substantially lower than the Peruvian level (Hong, 2023).

For the counterfactual experiment, I recalibrate  $\beta$ ,  $\chi_1$ , and  $\chi_2$  by targeting U.S. MPC moments and workers' aggregate wealth. For the target MPC moments, I estimate U.S. MPC at each earnings decile by applying Blundell et al. (2008)'s method to the 2005–2017 waves of the Panel Study of Income Dynamics (PSID). Since the reference period of the PSID data is a year, I obtain annual U.S. MPC estimates, while my HASOE model is a quarterly model. Given the frequency mismatch between the model and data, I target the annual MPC estimates as follows: I first simulate individual workers' quarterly earnings and consumption series from the model; then, I convert the quarterly series to annual series by summing them over every four quarters; using the simulated annual data, I compute the model counterparts of the MPC estimates by applying the same MPC estimation procedure applied to the PSID; I calibrate  $\beta$ ,  $\chi_1$ , and  $\chi_2$  such that the model counter-

<sup>52</sup>For impulse responses of  $Y$ ,  $I$ , and  $TB/Y$  in the HASOE ( $z, g, \mu, \eta$ ) model, see Fig M.2b in Online Appendix M.

<sup>53</sup>The impulse response of  $C^W$  in the second to ninth deciles changes gradually across deciles in response to each shock. The figures for the second to ninth deciles are readily available upon request.

parts are as close as possible to the MPC estimates.<sup>54</sup> For workers’ aggregate wealth, I target its value in the benchmark Peruvian economy ( $(A^W + B^W)/Y = 6.658$ , as reported in Table 3).

Table 7 reports the recalibrated parameter values. The value of  $\chi_1$  in the counterfactual economy (0.716) is markedly lower than that in the benchmark economy (6.694). This means that the U.S. MPC estimates discipline the model to exhibit weaker financial friction and thus lower MPC and weaker precautionary saving than the Peruvian MPC estimates do. The value of  $\beta$  in the counterfactual economy (0.974) is noticeably greater than that in the benchmark economy (0.948). This is because workers have a weaker precautionary saving motive in the counterfactual economy than in the benchmark economy and thus must be more patient to achieve the same amount of aggregate wealth. The average haircut rate ( $\chi_t^W / \int_{\Gamma} \int_{e_1, e_2, b_-, a_-} |v_{i,t}| d\Psi_t dG$ ) is 6.5% in the counterfactual economy, which is substantially lower than that in the benchmark economy (43.5%).

The joint recalibration again matches targets well, even though only three parameters are used to target eleven moments. First, workers’ aggregate wealth  $(A^W + B^W)/Y$  in the counterfactual economy is 6.564, which is close to the target, 6.658. Second, Figure 5a plots the annual MPC estimates obtained from the PSID (labeled ‘Data’) and their model counterparts in the counterfactual economy (labeled ‘Model’) and shows that the latter closely tracks the former.

Figure 5b compares the model-predicted quarterly MPCs in the benchmark and counterfactual economies and shows that there is a substantial MPC gap between the two economies: the mean quarterly MPC across deciles in the benchmark economy is 0.209, which is approximately twice as large as that in the counterfactual economy, 0.117. This figure suggests that when we interpret the Peruvian and U.S. MPC estimates reflecting the different reference periods of the underlying surveys through the lens of the HASOE model, the estimates tell us that Peruvian households exhibit substantially higher MPCs than U.S. households.<sup>55,56</sup>

Table 7: Recalibrated Parameters for the Counterfactual Economy

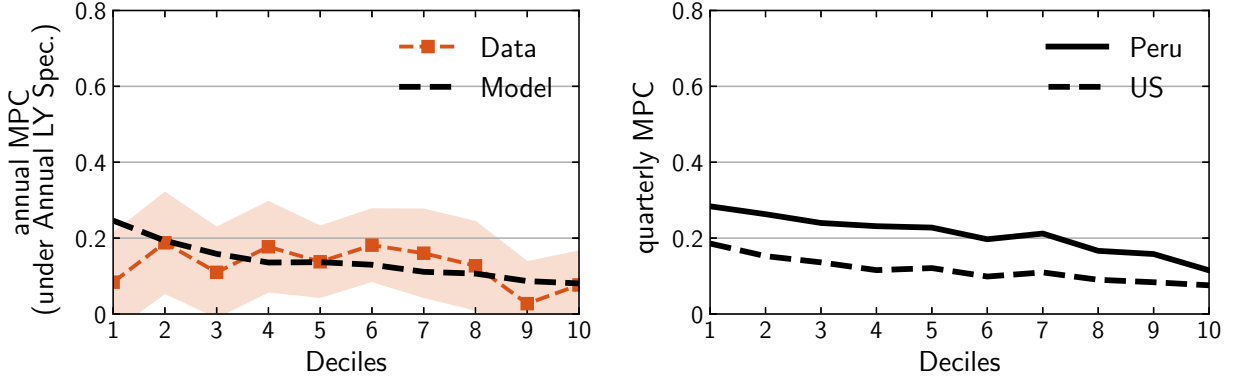
	Description	Value	Target / source
$\beta$	workers’ discount factor	0.974	} MPC estimates (from PSID) & $(A^W + B^W)/Y$ in Table 3
$\chi_1$	scale parameter for illiquid asset adjustment cost	0.716	
$\chi_2$	convexity parameter for illiquid asset adjustment cost	1.788	

<sup>54</sup>The same targeting method is used in Hong (2023) when comparing Peruvian and U.S. MPCs using a model.

<sup>55</sup>This result is consistent with the result of the model-based MPC comparison in Hong (2023), where I employ a standard one-asset incomplete-market model to interpret the MPC estimates reflecting different reference periods.

<sup>56</sup>Annual and quarterly MPCs mean a consumption response within a year and a quarter, respectively, after the realization of a shock. By definition, annual MPC should be greater than quarterly MPC. However, the annual MPC estimates (and their model counterparts) in Figure 5a are not much greater than the model-predicted quarterly MPC in Figure 5b. This is because the annual MPC estimates (and their model counterparts) in Figure 5a underestimate the true annual MPC in the model due to a ‘time aggregation problem’ noted by Crawley (2020): when households receive earnings shocks and make consumption decisions at a certain frequency while Blundell et al. (2008)’s method is applied to data aggregated over a longer time period, the method significantly underestimates the consumption





(a) U.S. Annual MPC Estimates: Data vs. Model      (b) Model-Predicted Quarterly MPCs: Peru vs. U.S.

Figure 5: MPCs in the Counterfactual Economy

*Notes:* Figure 5a plots the annual MPC estimates obtained from the PSID (labeled ‘Data’) and their model counterparts in the counterfactual economy (labeled ‘Model’). Shaded areas are 95% confidence intervals. Figure 5b plots the model-predicted quarterly MPCs in the benchmark and counterfactual economies (labeled ‘Peru’ and ‘US’, respectively).

Now, we are ready to examine the business cycle implications of this MPC gap. In Figure 6, I plot the posterior distributions of output volatility  $\sigma(\Delta \log Y_t)$ , consumption volatility  $\sigma(\Delta \log C_t)$ , and their ratio in the benchmark economy and compare them with the corresponding distributions in the counterfactual economy, which are obtained by evaluating the recalibrated model at each parameter draw from the benchmark economy’s posterior distribution. Figure 6a shows that the output volatility distribution is nearly identical between the two economies.<sup>57</sup> On the other hand, Figure 6b shows that the consumption volatility distribution changes substantially. The distribution in the counterfactual economy exhibits a lower mean (0.029) and greater standard deviation (0.006) than that in the benchmark economy (mean 0.036 and standard deviation 0.002). Figure 6c compares the distribution of the consumption-output volatility ratio. On average, the ratio is 1.215 in the benchmark economy (excess consumption volatility) and 0.993 in the counterfactual economy (absence of excess consumption volatility). Moreover, unlike in the benchmark economy where consumption is more volatile than output in nearly all (99.95%) of the posterior distribution, the excess consumption volatility disappears in the counterfactual economy in most parts (52.93%) of the posterior distribution, including the posterior median and mode.<sup>58</sup>

sensitivity to transitory shocks. See [Hong \(2023\)](#) for a detailed discussion.

<sup>57</sup>In the model, output is determined by firms’ Cobb–Douglas production,  $Y_t = z_t K_{t-1}^\alpha (X_t L_t)^{1-\alpha}$ . Because  $K_{t-1}$  is a slow-moving variable and  $L_t$  is determined by  $z_t$ ,  $X_t$ ,  $X_{t-1}$ , and  $K_{t-1}$  (through labor demand (A.16) in Online Appendix A.1 and labor supply (4)), aggregate shocks  $z_t$  and  $g_t$  almost entirely determine output volatility. Given this supply-side feature of the model, it is not surprising that the two economies exhibit similar output volatilities.

<sup>58</sup>At the posterior mode,  $\frac{\sigma(\Delta \log C_t)}{\sigma(\Delta \log Y_t)}$  is 1.232 and 0.746 in the benchmark and counterfactual economies, respectively.

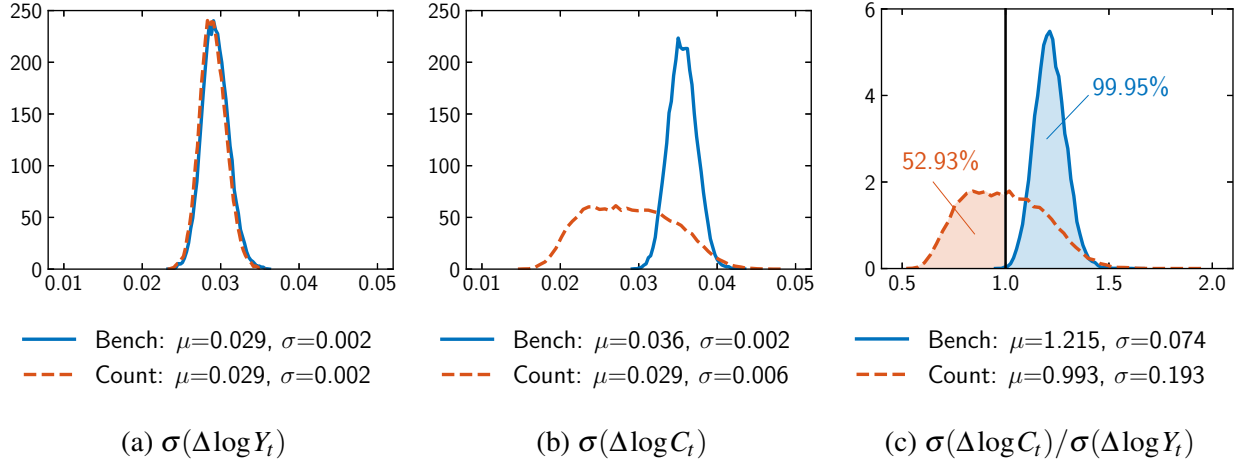


Figure 6: Consumption and Output Volatilities and Their Ratio: Benchmark vs. Counterfactual

*Notes:* Figures 6a, 6b, and 6c plot the posterior distributions of  $\sigma(\Delta \log Y_t)$ ,  $\sigma(\Delta \log C_t)$ , and  $\frac{\sigma(\Delta \log C_t)}{\sigma(\Delta \log Y_t)}$  in the benchmark economy, respectively (labeled ‘Bench’), and compare them with the corresponding distributions in the counterfactual economy (labeled ‘Count’), which are obtained by evaluating the recalibrated model at each parameter draw from the benchmark economy’s posterior distribution. The legend reports the mean and standard deviation of each distribution.

To understand the consumption volatility change, I compute consumption impulse responses in the counterfactual economy by evaluating the recalibrated model at each parameter draw from the benchmark economy’s posterior distribution. Then, I decompose the consumption responses into entrepreneurs’ responses and workers’ responses driven by  $w_t \bar{l}_t$ ,  $r_t^a$ ,  $r_t^b$ , and  $\eta$ , as in Figure 4 for the benchmark economy. Figure 7 plots the mean responses across the posterior distribution.

Figure 7a shows that the mean consumption response to a financial friction shock  $\eta$  is substantially weaker in the counterfactual economy than that in the benchmark economy (plotted in Figure 4a), and the weaker response is driven by workers’ weaker response to the change of  $\eta_t$  in their budget constraint. In the counterfactual economy, workers face weaker financial friction and also have a weaker precautionary saving motive than in the benchmark economy, and thus, heightened financial friction depresses workers’ consumption less intensely.

Figure 7b shows that the mean consumption response to a stationary productivity shock  $z$  is also substantially weaker in the counterfactual economy than in the benchmark economy (plotted in Figure 4b), and the weaker response is driven by workers’ weaker responses to  $w_t \bar{l}_t$  and  $r_t^a$  in their budget constraint. However, as Figure M.1 of Online Appendix M shows, the impulse responses of the drivers ( $w_t \bar{l}_t$  and  $r_t^a$ ) to a  $z$  shock are very similar between the two economies. This means that when a  $z$  shock is realized, workers face a similar degree of income fluctuations between the benchmark and counterfactual economies, but those in the counterfactual economy exhibit much smaller MPC and thus translate the income fluctuations far less into consumption fluctuations.

The weak consumption responses to  $\eta$  and  $z$  shocks observed in Figures 7a and 7b and the

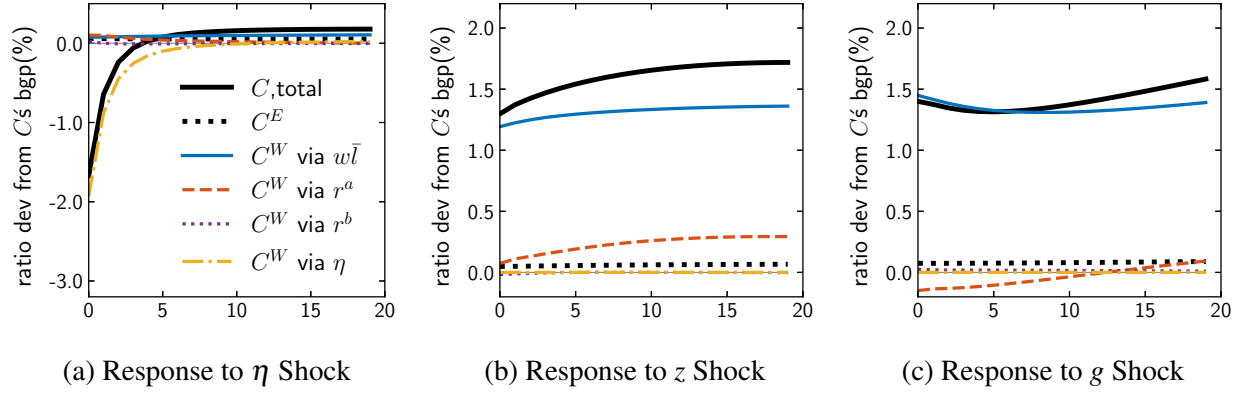


Figure 7: Consumption Response Decomposition in the Counterfactual Economy

*Notes:* Figures 7a, 7b, and 7c plot the consumption response to one-standard-deviation  $\eta$ ,  $z$ , and  $g$  shocks, respectively, in the counterfactual economy evaluated at the parameter draws from the benchmark economy's posterior distribution. In each figure, the consumption response is decomposed into entrepreneurs' response and workers' responses driven by  $w\bar{l}$ ,  $r^a$ ,  $r^b$ , and  $\eta$  in their budget constraints. The model statistics are computed at each posterior draw, and their means across the posterior distribution are plotted. The unit of the y-axis, 'ratio dev from  $C$ 's bgp(%)', represents the deviation from the balanced growth path (b.g.p) of the variable of interest divided by  $C_t$  on the b.g.p.

underlying economic mechanisms explain why the mean consumption volatility is smaller in the counterfactual economy than in the benchmark economy, as presented in Figure 6b.

Two other observations are worth noting regarding the counterfactual economy. First, unlike  $\eta$  and  $z$  shocks, a trend shock  $g$  generates more consumption fluctuations in the counterfactual economy than in the benchmark economy.<sup>59</sup> Figures 7c and 4c reveal the reason why: the consumption response to a  $g$  shock is stronger in the counterfactual economy than in the benchmark economy. This is because Aguiar and Gopinath (2007)'s permanent income effect is revived in the counterfactual economy due to weaker financial friction and precautionary saving behavior.

Second, although this section focuses on how the model prediction changes when the counterfactual economy is evaluated at the parameter draws from the benchmark economy's posterior distribution, it could also be of a separate interest to see how this counterfactual economy would explain emerging market business cycles if estimated. In Online Appendix K, I indeed estimate the counterfactual economy using the same data and methods as those used in the estimation of the benchmark economy. The estimated counterfactual economy generates the stylized patterns of emerging market business cycles reasonably well, including excess consumption volatility. Importantly, it does so in a very similar way as the RASOE model does: a trend shock generates most

<sup>59</sup>Using the variance decomposition technique, one can also decompose the change in consumption variance (from the benchmark to the counterfactual economy) into the changes originating from each shock. In terms of the posterior mean,  $\sigma^2(\Delta \log C_t)$  decreases by 32.2% in the counterfactual economy. Of this -32.2% change, -27.9%p and -18.0%p come from  $\eta$  and  $z$  shocks generating less fluctuations, respectively, while +13.7%p comes from a  $g$  shock generating more fluctuations.

of the consumption fluctuations. This result reconfirms the finding in the previous paragraph that [Aguiar and Gopinath \(2007\)](#)'s permanent income effect is revived in the counterfactual economy.

## 6 A Debate: What Drives Large Consumption Swings?

My paper is most closely related to [Guntin et al. \(2023\)](#). These two papers share a view that micro data, when interpreted through a heterogeneous-agent model, provide important information about what drives large consumption fluctuations. However, they come to different conclusions: [Guntin et al. \(2023\)](#) find that the permanent income effect of a trend shock drives large consumption swings, while I find that a financial friction shock and a stationary productivity shock mainly drive consumption fluctuations. In this sense, a longstanding debate on what drives consumption fluctuations in emerging economies, particularly between a trend shift and financial friction<sup>60</sup>, continues in the heterogeneous-agent open economy landscape. In this section, I discuss key differences between the two papers and how they come to different conclusions.

**Micro Moments.** The two papers use different information from micro data. [Guntin et al. \(2023\)](#) use a group-level consumption-income elasticity between the peak and trough around a crisis episode,  $\hat{\epsilon}_{GOP}^G := \frac{\log \bar{c}_{t+h}^G - \log \bar{c}_t^G}{\log \bar{y}_{t+h}^G - \log \bar{y}_t^G}$ , where  $\bar{c}_t^G$  and  $\bar{y}_t^G$  are the group-average (residualized) consumption and income, and  $t$  and  $t+h$  are the peak and trough around a crisis, respectively. I use an MPC out of idiosyncratic transitory income shocks obtained by using [Blundell et al. \(2008\)](#)'s method.

The two micro moments use different sources of income variation. [Guntin et al. \(2023\)](#)'s elasticity washes out all the idiosyncratic income risk by group-averaging consumption and income and exploits aggregate income risk borne differently by each group only. In contrast, my MPC moment exploits idiosyncratic income risk only.<sup>61</sup> Given that individual households face much greater idiosyncratic risk than aggregate risk<sup>62</sup>, the MPC moment may better capture household consumption smoothing disruption than the group-level elasticity.

**Models.** Both papers interpret micro moments using a heterogeneous-agent small open economy model, but the models exhibit two important differences. First, the aggregate precautionary saving stock is composed of liquid wealth in [Guntin et al. \(2023\)](#)'s model, while it is mostly composed of illiquid wealth in my model. As a result, households in my model face an expensive adjustment cost when they borrow from the future by cashing out their assets. Moreover, their precautionary

<sup>60</sup>For a trend shift, see [Aguiar and Gopinath \(2007\)](#). For financial friction, see [Neumeyer and Perri \(2005\)](#), [Garcia-Cicco et al. \(2010\)](#), [Chang and Fernández \(2013\)](#), [Mendoza \(2010\)](#), and [Bianchi \(2011\)](#).

<sup>61</sup>The effect of aggregate income risk is removed when extracting a predictable component of income and consumption, which includes a time fixed effect.

<sup>62</sup>To have a quantitative sense of their relative magnitude, one can compare the log growth dispersion between aggregate and idiosyncratic incomes.  $\sigma(\Delta \log e_{i,t})$  is 0.689 in ENAHO, where  $\log e_{i,t}$  is the unpredictable component of log earnings. This number is 25.2 times as large as  $\sigma(\Delta \log Y_t) = 0.027$ , where  $Y_t$  is aggregate income.

saving is enhanced because they are more concerned about a future low income path due to the financial friction. As discussed in subsection 3.4, these are the reasons why the consumption response to a trend shock is muted in my model. In [Guntin et al. \(2023\)](#)'s model, households do not face such an adjustment cost, and their precautionary saving is also not enhanced by it. Thus, a trend shock can generate a large consumption response without interruption in their model.

Second, unlike [Guntin et al. \(2023\)](#), I allow different types of shocks to be realized at different times in the model. When examining whether a model can explain a micro data pattern, [Guntin et al. \(2023\)](#) simulate a crisis by hitting the economy with a one-time, single-type shock. In the data, they find a flat or upward-sloping graph of their elasticity over income deciles (*i.e.*, higher-income households exhibit higher elasticity). In the model, they consider two scenarios: i) a trend shock hits the economy where households face constant borrowing constraints, and ii) a stationary productivity shock hits the economy where households face aggregate-income-dependent borrowing constraints. They find that the first scenario can explain the flat or upward-sloping elasticity graph, while the second scenario cannot, as it produces a downward-sloping graph. Based on these results, they conclude that a trend shock drives large consumption swings during a crisis.

My model indeed produces a qualitatively similar impulse response outcome to theirs: a financial friction shock hits lower income deciles more strongly, while a trend shock hits higher income deciles more intensely.<sup>63</sup> However, my model allows different types of shocks to hit the economy at different times and, as a result, depicts a quite different story about what happened during the 2008 Peruvian recession, an event that is one of the episodes that [Guntin et al. \(2023\)](#) study and is also within my macro data period. Importantly, I obtain an upward-sloping graph of [Guntin et al. \(2023\)](#)'s elasticity in my model although heightened financial friction still plays a major role in the story, suggesting that the upward-sloping graph might not necessarily favor trend shift theory over financial friction theory.

To see how my model depicts the 2008 Peruvian recession, I smooth aggregate shocks at the posterior mode.<sup>64</sup> As the first two panels of Figure 8 show, the simulated output and consumption using smoothed shocks in the model closely track the data counterparts in 2007–2010.<sup>65</sup> The third panel plots smoothed workers' consumption, showing similar dynamics as total consumption. Using the smoothed shocks, I compute [Guntin et al. \(2023\)](#)'s elasticity around the recession.<sup>66</sup>

<sup>63</sup>See the discussion at the end of section 4 as well as Figure M.3 of Online Appendix M.

<sup>64</sup>See Online Appendix L for details. I thank Nils Gornemann for suggesting the smoothing analysis.

<sup>65</sup>Figure L.1 in Online Appendix L plots all the simulated observable variables using smoothed shocks in the model and their data counterparts throughout the data period (1980–2018), showing that the model and data track each other very closely. They are not exactly the same only because measurement errors are not included in the simulation.

<sup>66</sup>Some details regarding the elasticity calculation are noteworthy: i) my model is evaluated at the posterior mode; ii) the elasticity is measured for a synthetic group (*i.e.*, not for a fixed group), as in [Guntin et al. \(2023\)](#); iii) earnings are used as the income measure, following [Guntin et al. \(2023\)](#)'s treatment of the ENAHO data; and iv) the sample is composed of workers only, reflecting [Guntin et al. \(2023\)](#)'s sample selection where only observations reporting positive income (which is positive earnings observed in ENAHO in their analysis) are used.

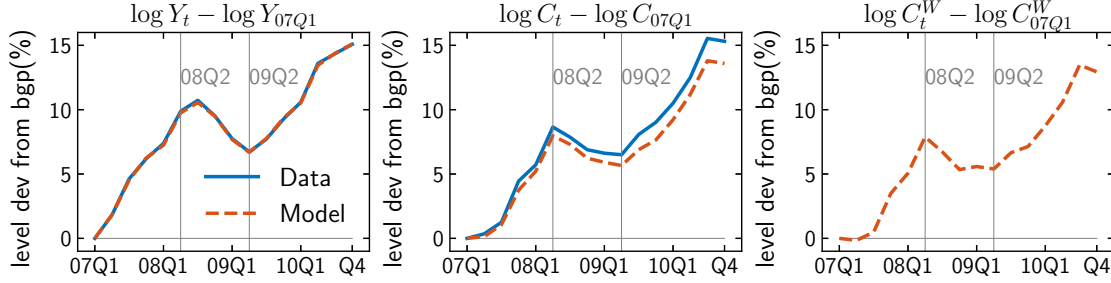


Figure 8: What Happens in 2007–2010, Peru

*Notes:* The first two panels plot i) output and consumption from national accounts in logs after seasonal adjustment and log-linear detrending (labeled ‘Data’) and ii) their model counterparts simulated using smoothed shocks at the posterior mode (labeled ‘Model’). The third panel plots the smoothed average workers’ consumption.

Based on the consumption and output dynamics in Figure 8, I define the peak and trough as 2008Q2 and 2009Q2, respectively (which are indicated by gray vertical lines).<sup>67</sup> Figure 9 plots the elasticity at each earnings decile, exhibiting an upward-sloping graph.

Figures 10 and 11 tell a more detailed story about what happened during the recession according to the simulation with smoothed shocks. As Figure 10 shows, a large financial friction shock ( $\eta$ ) hits the economy first in 2008Q3, while productivities ( $z$  and  $g$ ) remain stable. Afterwards, large  $z$  and  $g$  shocks hit the economy in the following quarters (2008Q4 and 2009Q1, respectively). The financial friction ( $\eta$ ) is at its peak in 2008Q4 and then nearly returns to a precrisis level in 2009Q1, while productivities are still very low.<sup>68</sup> The first panel in Figure 11 shows how

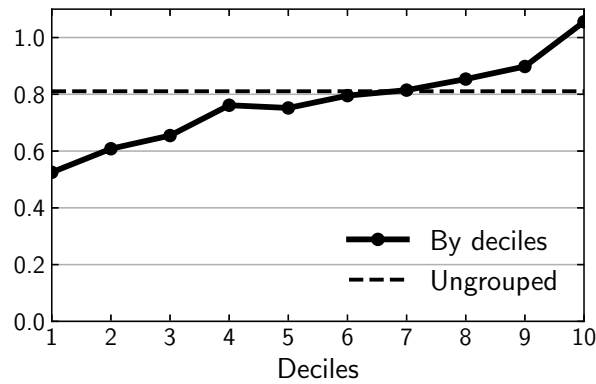


Figure 9: Guntin et al. (2023)’s Group-Level Consumption-Income Elasticity in My Model

<sup>67</sup>Guntin et al. (2023) identify the peak and trough around the 2008 Peruvian recession as 2007 and 2010, respectively, based on the aggregated individual consumption from ENAHO rather than based on national accounts. This identification can be affected by time-varying measurement errors in the survey. Indeed, the first two panels of Figure 8 plot the output and consumption in national accounts and suggest that the Peruvian economy was not in the trough during 2010. Economic Commission for Latin America and the Caribbean (2010) shares this view by noting on page 79 that “[t]he Peruvian economy was strong in 2010, driven by growing domestic demand.”

<sup>68</sup>Gilchrist and Zakrajšek (2012) empirically find that a financial disruption leads a slowdown in real activities.

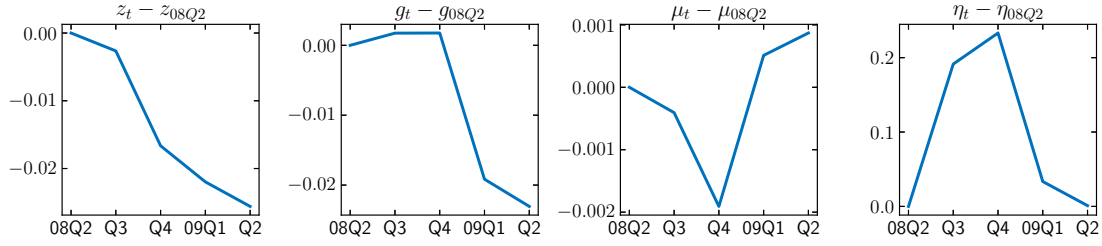


Figure 10: Smoothed Shocks around the 2008 Peruvian Recession

these different shocks at different times drive consumption fluctuations during the recession. The heightened financial friction ( $\eta$ ) drives a consumption plunge in 2008Q3. Afterwards, the stationary productivity ( $z$ ) further drags down consumption in 2008Q4, and both productivities ( $z$  and  $g$ ) maintain consumption at a depressed level in 2009Q1-Q2 despite alleviated financial friction. The consumption recovery due to alleviated financial friction in 2009Q1-Q2 itself goes beyond the precrisis level because of an expectation that the financial condition will be favorable for a while.

The second and third panels of Figure 11 show how these shocks affect the group-average consumption of the bottom and top deciles differently. The heightened financial friction ( $\eta$ ) in 2008Q3 generates a disproportionately large consumption plunge in the bottom decile, while the top decile consumption barely responds to it. The consumption recovery due to alleviated financial friction in 2009Q1-Q2 is also much stronger in the bottom decile than in the top decile. The stationary productivity ( $z$ ) drags consumption down during 2008Q4-2009Q2 to a similar degree between the top and bottom deciles, while the nonstationary productivity ( $g$ ) drags consumption down during 2009Q1-Q2 more strongly in the top decile than in the bottom decile.<sup>69</sup>

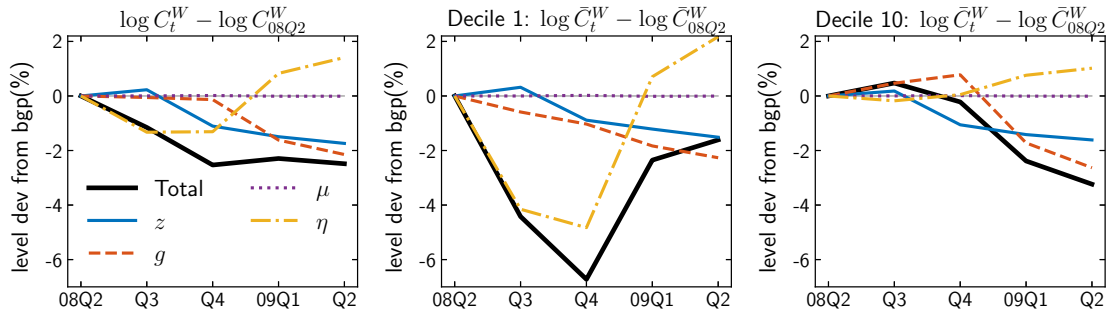


Figure 11: Decomposition of Smoothed Workers' Consumption Fluctuations across Shocks

*Notes:* The three panels in Figure 11 plot the smoothed average workers' consumption fluctuations in the whole economy and in the bottom and top earnings deciles and decompose them into fluctuations driven by each shock.

Although their finding is for the U.S. economy, this empirical pattern is consistent with the story my model delivers about the 2008 Peruvian recession.

<sup>69</sup>The decomposition of consumption fluctuations in the second to ninth deciles changes gradually across deciles. The figures for the decomposition in the second to ninth deciles are readily available upon request.



In short, a financial friction shock hits the economy in the early part of the crisis, strongly affecting lower deciles; in the later part of the crisis, the financial friction attenuates to a precrisis level, while productivities slow down, affecting higher deciles more. Therefore, in terms of a (log) consumption change between the peak (08Q2) and trough (09Q2), lower deciles experience a smaller change than higher deciles. This is what [Guntin et al. \(2023\)](#)’s elasticity captures. However, it misses a large consumption swing driven by heightened financial friction and borne disproportionately more by lower deciles in the early part of the crisis.

## 7 Conclusion

This paper explains emerging market business cycles using a HASOE model with two-asset household heterogeneity over liquid and illiquid assets, disciplined by MPC estimates from Peruvian micro data. I find that a conventional mechanism through which the corresponding RASOE model explains emerging market business cycles does not work in the HASOE model because it is hindered by strong financial friction and precautionary saving. Instead, the HASOE model explains emerging market business cycles through a new mechanism in which high MPC and correspondingly strong financial friction and precautionary saving play important roles. When household MPCs are adjusted to the U.S. level, which is substantially lower than the Peruvian level, excess consumption volatility disappears in most parts of the posterior distribution.

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## [Online Appendix]

# Emerging Market Business Cycles with Heterogeneous Agents

Seungki Hong

## A Details of the HASOE Model

### A.1 Equilibrium under Deterministic Paths of Aggregate Shocks

This subsection characterizes the equilibrium conditions of the HASOE model in section 4 when the economy faces deterministic paths of aggregate shocks  $\{z_t, g_t, \mu_t, \eta_t\}_{t=0}^{\infty}$ . The HASOE model in subsection 2.1 has the same equilibrium conditions except that  $\eta_t$  is replaced with 1.

Workers' problem can be expressed as the following Bellman equation.

$$V_t^W(e_1, e_2, b_-, a_-; \Gamma) = \max_{c, b, a} \frac{c^{1-\gamma}}{1-\gamma} + \beta \sum_{e'_1, e'_2} P(e'_1, e'_2 | e_1, e_2) V_{t+1}^W(e'_1, e'_2, b, a; \Gamma)$$

$$s.t. \quad c + b + a + \eta_t \chi_t(a - (1 + r_t^a) a_-, a_-; \Gamma) = w_t \Gamma e \bar{l}_t + (1 - \xi)(1 + r_t^b) b_- + (1 + r_t^a) a_-,$$

$$a \geq 0, \quad b \geq 0, \quad \text{and} \quad \log e = \log e_1 + \log e_2.$$

On the balanced growth path,  $V_t^W$  grows at the rate of  $(g^*)^{1-\gamma}$  or, equivalently,  $V_{t+1}^W = (g^*)^{1-\gamma} V_t^W$ .

Under the parametrization of  $\chi_t(v, a_-; \Gamma)$  in subsection 2.1, its first-order derivatives  $\chi_{1,t}(v, a_-; \Gamma)$  and  $\chi_{2,t}(v, a_-; \Gamma)$  are continuous in  $(v, a_-)$  everywhere, including an area around  $v = 0$ . Thus,  $\chi_t(v, a_-; \Gamma)$  is differentiable everywhere. Workers' optimality conditions are derived as follows.

$$V_t^W(e_1, e_2, b_-, a_-; \Gamma) = \max_{c, b, a} \frac{c^{1-\gamma}}{1-\gamma} + \beta \sum_{e'_1, e'_2} P(e'_1, e'_2 | e_1, e_2) V_{t+1}^W(e'_1, e'_2, b, a; \Gamma)$$

$$+ \lambda \{w_t \Gamma e \bar{l}_t + (1 - \xi)(1 + r_t^b) b_- + (1 + r_t^a) a_- - c - b - a - \eta_t \chi_t(a - (1 + r_t^a) a_-, a_-; \Gamma)\} + \varphi^b b + \varphi^a a.$$

$$\lambda = c^{-\gamma}, \tag{A.1}$$

$$\lambda = \beta \sum_{e'_1, e'_2} P(e'_1, e'_2 | e_1, e_2) V_{b,t+1}^W(e'_1, e'_2, b, a; \Gamma) + \varphi^b, \tag{A.2}$$

$$\lambda \{1 + \eta_t \chi_{1,t}(a - (1 + r_t^a) a_-, a_-; \Gamma)\} = \beta \sum_{e'_1, e'_2} P(e'_1, e'_2 | e_1, e_2) V_{a,t+1}^W(e'_1, e'_2, b, a; \Gamma) + \varphi^a, \tag{A.3}$$

$$V_{b,t}^W(e_1, e_2, b_-, a_-; \Gamma) = (1 - \xi)(1 + r_t^b) \lambda, \tag{A.4}$$

$$V_{a,t}^W(e_1, e_2, b_-, a_-; \Gamma) = \lambda \{(1 + r_t^a) + (1 + r_t^a) \eta_t \chi_{1,t}(a - (1 + r_t^a) a_-, a_-; \Gamma) - \eta_t \chi_{2,t}(a - (1 + r_t^a) a_-, a_-; \Gamma)\}, \tag{A.5}$$

$$c + b + a + \eta_t \chi_t(a - (1 + r_t^a) a_-, a_-; \Gamma) = w_t \Gamma e \bar{l}_t + (1 - \xi)(1 + r_t^b) b_- + (1 + r_t^a) a_-, \tag{A.6}$$

$$\varphi^b \geq 0, \quad b \geq 0, \quad \varphi^b b = 0, \quad \text{and} \tag{A.7}$$

$$\varphi^a \geq 0, \quad a \geq 0, \quad \varphi^a a = 0. \tag{A.8}$$

Entrepreneurs' optimality conditions for their problem (6) are characterized as follows.

$$C_t^E + A_t^E = R_t^E + (1 + r_t^a)A_{t-1}^E, \quad t \geq 0, \quad \text{and} \quad (\text{A.9})$$

$$(C_t^E)^{-\gamma} = \beta_E(1 + r_{t+1}^a)(C_{t+1}^E)^{-\gamma}, \quad t \geq 0. \quad (\text{A.10})$$

Firms' optimality conditions for their problem (11) are characterized as follows.

$$\Pi_t = Y_t - w_t L_t - I_t - \Phi(K_t, K_{t-1}) + F_t - (1 + r_{t-1})F_{t-1}, \quad (\text{A.11})$$

$$Y_t = z_t K_{t-1}^\alpha (X_t L_t)^{1-\alpha}, \quad (\text{A.12})$$

$$I_t = K_t - (1 - \delta)K_{t-1}, \quad (\text{A.13})$$

$$(1 + r_{t+1}^a) \left\{ 1 + \phi \left( \frac{K_t}{K_{t-1}} - g^* \right) \right\} = \alpha z_{t+1} \left( \frac{K_t}{X_{t+1} L_{t+1}} \right)^{\alpha-1} \quad (\text{A.14})$$

$$+ \left\{ 1 - \delta + \phi \left( \frac{K_{t+1}}{K_t} - g^* \right) \frac{K_{t+1}}{K_t} - \frac{\phi}{2} \left( \frac{K_{t+1}}{K_t} - g^* \right)^2 \right\}, \quad t \geq 0, \quad (\text{A.15})$$

$$1 + r_{t+1}^a = 1 + r_t, \quad t \geq 0, \quad \text{and} \quad (\text{A.15})$$

$$w_t = (1 - \alpha) z_t X_t \left( \frac{K_{t-1}}{X_t L_t} \right)^\alpha. \quad (\text{A.16})$$

Given the initial conditions on  $\Psi_0(e_1, e_2, b_-, a_- | \Gamma)$ ,  $X_{-1}$ ,  $A_{-1}$ ,  $A_{-1}^E$ ,  $K_{-1}$ ,  $D_{-1}$ ,  $B_{-1}$ ,  $F_{-1}$ , and  $r_{-1}$  and deterministic paths of aggregate shocks  $\{z_t, g_t, \mu_t, \eta_t\}_{t=0}^\infty$ , (i) individual workers' policy functions  $\{c_t(e_1, e_2, b_-, a_-; \Gamma), b_t(e_1, e_2, b_-, a_-; \Gamma), a_t(e_1, e_2, b_-, a_-; \Gamma)\}_{t=0}^\infty$ , first-order derivatives of the value functions  $\{V_{b,t}^W(e_1, e_2, b_-, a_-; \Gamma), V_{a,t}^W(e_1, e_2, b_-, a_-; \Gamma)\}_{t=0}^\infty$ , and Lagrangian multipliers  $\{\lambda_t(e_1, e_2, b_-, a_-; \Gamma), \phi_t^b(e_1, e_2, b_-, a_-; \Gamma), \phi_t^a(e_1, e_2, b_-, a_-; \Gamma)\}_{t=0}^\infty$  that satisfy workers' optimality conditions (A.1) - (A.8), (ii) conditional cumulative distributions  $\{\Psi_t(e_1, e_2, b_-, a_- | \Gamma)\}_{t=1}^\infty$  that evolve over time according to equation (5), and (iii) prices and aggregate variables  $\{r_t^b, r_t^a, r_t, w_t, q_t, \bar{l}_t, L_t, \Pi_t, Y_t, I_t, K_t, F_t, D_t, TB_t, C_t, C_t^E, A_t, A_t^E, R_t^E, B_t, \chi_t^{agg}\}_{t=0}^\infty$  satisfying entrepreneurs' optimality conditions (A.9) and (A.10), firms' optimality conditions (A.11) - (A.16), aggregation equations (7), (8), (9), and (22), and other equilibrium conditions (3), (4), (12), (13), (15), (16), (17), (18), and (20) constitute the equilibrium of the economy.

By Walras' law, the following resource constraint holds in the economy.

$$C_t + I_t + \frac{\phi}{2} \left( \frac{K_t}{K_{t-1}} - g^* \right)^2 K_{t-1} = Y_t + D_t - (1 + r_{t-1})D_{t-1}. \quad (\text{A.17})$$

## A.2 Detrended and Normalized Equilibrium under Deterministic Paths of Aggregate Shocks

Since the equilibrium is nonstationary due to the stochastic trend  $\{X_t\}_{t=0}^\infty$ , we need to detrend the equilibrium to make it stationary. I detrend the variables and functions as follows.

$$\tilde{x}_t := x_t / X_{t-1}, \quad x_t = c_{i,t}, v_{i,t}, Y_t, C_t, C_t^E, C_t^W, I_t, R_t^E, \chi_t^{agg}, \chi_t^W, w_t, \Pi_t, TB_t,$$

$$\tilde{x}_t := x_t / X_t, \quad x_t = b_{i,t}, a_{i,t}, B_t, B_t^W, A_t, A_t^E, A_t^W, q_t, D_t, K_t, F_t,$$



$$\begin{aligned}
\tilde{x}_t &:= x_t / (X_{t-1})^{-\gamma}, \quad x_t = \lambda_{i,t}, \varphi_{i,t}^b, \varphi_{i,t}^a, \\
\tilde{f}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_- | \Gamma) &:= f_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1} | \Gamma), \quad f_t = \Psi_t, \\
\tilde{f}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-; \Gamma) &:= f_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1}; \Gamma) / X_{t-1}, \quad f_t = c_t, \\
\tilde{f}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-; \Gamma) &:= f_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_t; \Gamma) / X_t, \quad f_t = b_t, a_t, \\
\tilde{f}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-; \Gamma) &:= f_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1}; \Gamma) / X_{t-1}^{-\gamma}, \quad f_t = V_{b,t}^W, V_{a,t}^W, \lambda_t, \varphi_t^a, \varphi_t^b, \quad \text{and} \\
\tilde{\chi}_t(\tilde{v}, \tilde{a}_-; \Gamma) &:= \chi_t(\tilde{v} X_{t-1}, \tilde{a}_- X_{t-1}; \Gamma) / X_{t-1} = \chi_1 \left| \frac{\tilde{v}}{(1+r_t^a) \tilde{a}_- + \chi_0 \Upsilon(\Gamma)} \right|^{\chi_2} ((1+r_t^a) \tilde{a}_- + \chi_0 \Upsilon(\Gamma)),
\end{aligned}$$

where  $\Upsilon(\Gamma) = \tilde{w}_{ss} \Gamma \bar{e} \bar{l}_{ss}$  and  $\tilde{w}_{ss}$  and  $\bar{l}_{ss}$  are the steady state values of  $\tilde{w}_t$  and  $\bar{l}_t$ , respectively.

Moreover, workers' problem can be normalized such that  $\Gamma$  is irrelevant as follows.

$$\begin{aligned}
\hat{x}_t &:= \tilde{x}_t / \Upsilon(\Gamma), \quad \tilde{x}_t = \tilde{c}_{i,t}, \tilde{v}_{i,t}, \tilde{b}_{i,t}, \tilde{a}_{i,t}, \tilde{C}_t^W, \tilde{B}_t^W, \tilde{A}_t^W, \tilde{\chi}_t^W, \\
\hat{x}_t &:= \tilde{x}_t / \Upsilon(\Gamma)^{-\gamma}, \quad \tilde{x}_t = \tilde{\lambda}_{i,t}, \tilde{\varphi}_{i,t}^b, \tilde{\varphi}_{i,t}^a, \\
\hat{w}_t &:= \tilde{w}_t / \tilde{w}_{ss}, \quad \hat{l}_t := \bar{l}_t / \bar{l}_{ss}, \quad \hat{e}_{i,t} := e_{i,t} / \bar{e}, \quad \hat{e}_{1,i,t} := e_{1,i,t} / E[e_{1,i,t}], \quad \hat{e}_{2,i,t} := e_{2,i,t} / E[e_{2,i,t}], \\
\hat{f}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) &:= \tilde{f}(\tilde{e}_1 \hat{e}_1, \tilde{e}_2 \hat{e}_2, \Upsilon(\Gamma) \hat{b}_-, \Upsilon(\Gamma) \hat{a}_- | \Gamma), \quad \tilde{f}_t = \tilde{\Psi}_t \\
\hat{f}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) &:= \tilde{f}(\tilde{e}_1 \hat{e}_1, \tilde{e}_2 \hat{e}_2, \Upsilon(\Gamma) \hat{b}_-, \Upsilon(\Gamma) \hat{a}_-; \Gamma) / \Upsilon(\Gamma), \quad \tilde{f}_t = \tilde{c}_t, \tilde{b}_t, \tilde{a}_t, \\
\hat{f}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) &:= \tilde{f}(\tilde{e}_1 \hat{e}_1, \tilde{e}_2 \hat{e}_2, \Upsilon(\Gamma) \hat{b}_-, \Upsilon(\Gamma) \hat{a}_-; \Gamma) / \Upsilon(\Gamma)^{-\gamma}, \quad \tilde{f}_t = \tilde{V}_{b,t}^W, \tilde{V}_{a,t}^W, \tilde{\lambda}_t, \tilde{\varphi}_t^b, \tilde{\varphi}_t^a, \quad \text{and} \\
\hat{\chi}_t(\hat{v}, \hat{a}_-) &:= \tilde{\chi}_t(\tilde{v} \Upsilon(\Gamma), \tilde{a}_- \Upsilon(\Gamma); \Gamma) / \Upsilon(\Gamma) = \chi_1 \left| \frac{\hat{v}}{(1+r_t^a) \hat{a}_- + \chi_0} \right|^{\chi_2} ((1+r_t^a) \hat{a}_- + \chi_0).
\end{aligned}$$

Workers' detrended and normalized optimality conditions are characterized as follows.

$$\hat{\lambda} = \hat{c}^{-\gamma}, \quad (\text{A.18})$$

$$\hat{\lambda} = \beta g_t^{-\gamma} \sum_{\hat{e}'_1, \hat{e}'_2} P(\hat{e}'_1, \hat{e}'_2 | \hat{e}_1, \hat{e}_2) \hat{V}_{b,t+1}^W(\hat{e}'_1, \hat{e}'_2, \hat{b}, \hat{a}) + \hat{\varphi}^b, \quad (\text{A.19})$$

$$\hat{\lambda} \{1 + \eta_t \hat{\chi}_{1,t}(g_t \hat{a} - (1+r_t^a) \hat{a}_-, \hat{a}_-)\} = \beta g_t^{-\gamma} \sum_{\hat{e}'_1, \hat{e}'_2} P(\hat{e}'_1, \hat{e}'_2 | \hat{e}_1, \hat{e}_2) \hat{V}_{a,t+1}^W(\hat{e}'_1, \hat{e}'_2, \hat{b}, \hat{a}) + \hat{\varphi}^a, \quad (\text{A.20})$$

$$\hat{V}_{b,t}^W(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) = (1 - \xi)(1 + r_t^b) \hat{\lambda}, \quad (\text{A.21})$$

$$\begin{aligned}
\hat{V}_{a,t}^W(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) &= \hat{\lambda} \{ (1 + r_t^a) + (1 + r_t^a) \eta_t \hat{\chi}_{1,t}(g_t \hat{a} - (1 + r_t^a) \hat{a}_-, \hat{a}_-) \\
&\quad - \eta_t \hat{\chi}_{2,t}(g_t \hat{a} - (1 + r_t^a) \hat{a}_-, \hat{a}_-) \}, \quad (\text{A.22})
\end{aligned}$$

$$\hat{c} + g_t \hat{b} + g_t \hat{a} + \eta_t \hat{\chi}_t(g_t \hat{a} - (1 + r_t^a) \hat{a}_-, \hat{a}_-) = \hat{w}_t \hat{e} \hat{l}_t + (1 - \xi)(1 + r_t^b) \hat{b}_- + (1 + r_t^a) \hat{a}_-, \quad (\text{A.23})$$

$$\hat{\varphi}^b \geq 0, \quad \hat{b} \geq 0, \quad \hat{\varphi}^b \hat{b} = 0, \quad \text{and} \quad (\text{A.24})$$

$$\hat{\varphi}^a \geq 0, \quad \hat{a} \geq 0, \quad \hat{\varphi}^a \hat{a} = 0. \quad (\text{A.25})$$

The law of motion for  $\hat{\Psi}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)$  is characterized as follows.

$$\begin{aligned}
\hat{\Psi}_{t+1}(\hat{e}'_1, \hat{e}'_2, \hat{b}, \hat{a}) &= \int_{\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-} [P(\hat{e}_{1,t+1} \leq \hat{e}'_1 | \hat{e}_{1,t} = \hat{e}_1) P(\hat{e}_{2,t+1} \leq \hat{e}'_2) \\
&\quad I_{\{\hat{b}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) \leq \hat{b}, \hat{a}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) \leq \hat{a}\}}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)] d\hat{\Psi}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-). \quad (\text{A.26})
\end{aligned}$$

Entrepreneurs' detrended optimality conditions are characterized as follows.

$$\tilde{C}_t^E + g_t \tilde{A}_t^E = \tilde{R}_t^E + (1 + r_t^a) \tilde{A}_{t-1}^E, \quad t \geq 0, \quad \text{and} \quad (\text{A.27})$$

$$(\tilde{C}_t^E)^{-\gamma} = g_t^{-\gamma} \beta_E (1 + r_{t+1}^a) (\tilde{C}_{t+1}^E)^{-\gamma}, \quad t \geq 0. \quad (\text{A.28})$$

Firms' detrended optimality conditions are characterized as follows.

$$\tilde{\Pi}_t = \tilde{Y}_t - \tilde{w}_t L_t - \tilde{I}_t - \frac{\phi}{2} \left( \frac{\tilde{K}_t}{\tilde{K}_{t-1}} g_t - g^* \right)^2 \tilde{K}_{t-1} + g_t \tilde{F}_t - (1 + r_{t-1}) \tilde{F}_{t-1}, \quad t \geq 0, \quad (\text{A.29})$$

$$\tilde{Y}_t = z_t g_t^{1-\alpha} \tilde{K}_{t-1}^\alpha L_t^{1-\alpha}, \quad t \geq 0, \quad (\text{A.30})$$

$$\tilde{I}_t = g_t \tilde{K}_t - (1 - \delta) \tilde{K}_{t-1}, \quad t \geq 0, \quad (\text{A.31})$$

$$(1 + r_t) \left\{ 1 + \phi \left( \frac{\tilde{K}_t}{\tilde{K}_{t-1}} g_t - g^* \right) \right\} = \alpha z_{t+1} g_{t+1}^{1-\alpha} \left( \frac{\tilde{K}_t}{L_{t+1}} \right)^{\alpha-1} + \left\{ 1 - \delta + \phi \left( \frac{\tilde{K}_{t+1}}{\tilde{K}_t} g_{t+1} - g^* \right) \frac{\tilde{K}_{t+1}}{\tilde{K}_t} g_{t+1} - \frac{\phi}{2} \left( \frac{\tilde{K}_{t+1}}{\tilde{K}_t} g_{t+1} - g^* \right)^2 \right\}, \quad t \geq 0, \quad (\text{A.32})$$

$$1 + r_{t+1}^a = 1 + r_t, \quad t \geq 0, \quad \text{and} \quad (\text{A.33})$$

$$\tilde{w}_t = (1 - \alpha) z_t g_t^{1-\alpha} \left( \frac{\tilde{K}_{t-1}}{L_t} \right)^\alpha, \quad t \geq 0. \quad (\text{A.34})$$

The aggregation equations (7), (8), (9), and (22) are detrended and normalized as follows.

$$\tilde{C}_t = p \tilde{C}_t^W + (1 - p) \tilde{C}_t^E, \quad \tilde{C}_t^W = \Upsilon(\bar{\Gamma}) \hat{C}_t^W, \quad \hat{C}_t^W = \int_{\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-} \hat{c}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) d\hat{\Psi}_t. \quad (\text{A.35})$$

$$\tilde{A}_t = p \tilde{A}_t^W + (1 - p) \tilde{A}_t^E, \quad \tilde{A}_t^W = \Upsilon(\bar{\Gamma}) \hat{A}_t^W, \quad \hat{A}_t^W = \int_{\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-} \hat{a}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) d\hat{\Psi}_t. \quad (\text{A.36})$$

$$\tilde{B}_t = p \tilde{B}_t^W, \quad \tilde{B}_t^W = \Upsilon(\bar{\Gamma}) \hat{B}_t^W, \quad \hat{B}_t^W = \int_{\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-} \hat{b}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) d\hat{\Psi}_t. \quad (\text{A.37})$$

$$\tilde{\chi}_t^{agg} = p \tilde{\chi}_t^W, \quad \tilde{\chi}_t^W = \Upsilon(\bar{\Gamma}) \hat{\chi}_t^W, \quad \hat{\chi}_t^W = \int_{\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-} \eta_t \hat{\chi}_t(g_t \hat{a}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) - (1 + r_t^a) \hat{a}_-, \hat{a}_-) d\hat{\Psi}_t. \quad (\text{A.38})$$

Equations (4), (12), (13), (15), (17), (18), and (20) are detrended as follows.

$$\tilde{w}_t (p \bar{\Gamma} \bar{e})^{1+\omega} = \kappa L_t^\omega, \quad t \geq 0, \quad (\text{A.39})$$

$$\tilde{A}_t = \tilde{q}_t, \quad t \geq 0, \quad (\text{A.40})$$

$$1 + r_t^a = \frac{\tilde{\Pi}_t + g_t \tilde{q}_t}{\tilde{q}_{t-1}}, \quad t \geq 0, \quad (\text{A.41})$$

$$\tilde{F}_t = \tilde{D}_t + \tilde{B}_t, \quad t \geq 0, \quad (\text{A.42})$$

$$(1 - p) \tilde{R}_t^E = \xi (1 + r_t^b) \tilde{B}_{t-1} + \tilde{\chi}_t^{agg}, \quad (\text{A.43})$$

$$r_t = r^* + \psi \left\{ \exp \left( \frac{\tilde{D}_t - \tilde{D}^*}{\tilde{Y}^*} \right) - 1 \right\} - \theta_z (z_t - 1) - \theta_g \left( \frac{g_t}{g^*} - 1 \right) + \mu_t - 1, \quad t \geq 0, \quad \text{and} \quad (\text{A.44})$$

$$\tilde{T} \tilde{B}_t = -g_t \tilde{D}_t + (1 + r_{t-1}) \tilde{D}_{t-1}, \quad t \geq 0. \quad (\text{A.45})$$

Given the initial conditions on  $\hat{\Psi}_0(e_1, e_2, \tilde{b}_-, \tilde{a}_-)$ ,  $\tilde{A}_{-1}$ ,  $\tilde{A}_{-1}^E$ ,  $\tilde{K}_{-1}$ ,  $\tilde{D}_{-1}$ ,  $\tilde{B}_{-1}$ ,  $\tilde{F}_{-1}$ , and  $r_{-1}$  and

deterministic paths of aggregate shocks  $\{z_t, g_t, \mu_t, \eta_t\}_{t=0}^\infty$ , (i) individual workers' detrended and normalized policy functions  $\{\hat{c}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-), \hat{b}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-), \hat{a}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)\}_{t=0}^\infty$ , first-order derivatives of the detrended and normalized value functions  $\{\hat{V}_{b,t}^W(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-), \hat{V}_{a,t}^W(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)\}_{t=0}^\infty$ , and detrended and normalized Lagrangian multipliers  $\{\hat{\lambda}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-), \hat{\phi}_t^b(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-), \hat{\phi}_t^a(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)\}_{t=0}^\infty$  that satisfy workers' detrended and normalized optimality conditions (A.18) - (A.25), (ii) cumulative distributions  $\{\hat{\Psi}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)\}_{t=1}^\infty$  that evolve over time according to equation (A.26), and (iii) prices and aggregate variables  $\{r_t^b, r_t^a, r_t, \tilde{w}_t, \tilde{q}_t, \tilde{l}_t, L_t, \tilde{\Pi}_t, \tilde{Y}_t, \tilde{I}_t, \tilde{K}_t, \tilde{F}_t, \tilde{D}_t, \tilde{T}B_t, \tilde{C}_t, \tilde{C}_t^E, \tilde{A}_t, \tilde{A}_t^E, \tilde{R}_t^E, \tilde{B}_t, \tilde{\chi}_t^{agg}\}_{t=0}^\infty$  satisfying entrepreneurs' detrended optimality conditions (A.27) and (A.28), firms' detrended optimality conditions (A.29) - (A.34), detrended aggregation equations (A.35) - (A.38), and other detrended equilibrium conditions (3), (16), and (A.39) - (A.45) constitute the detrended equilibrium of the economy.

The resource constraint (A.17) is detrended as follows.

$$\tilde{C}_t + \tilde{I}_t + \frac{\phi}{2} \left( \frac{\tilde{K}_t}{\tilde{K}_{t-1}} g_t - g^* \right)^2 \tilde{K}_{t-1} = \tilde{Y}_t + g_t \tilde{D}_t - (1 + r_{t-1}) \tilde{D}_{t-1}, \quad t \geq 0. \quad (\text{A.46})$$

## B Details of the RASOE Model

### B.1 Equilibrium under Deterministic Paths of Aggregate Shocks

This subsection characterizes the equilibrium conditions of the RASOE model in subsection 2.2 when the economy is subject to deterministic paths of aggregate shocks  $\{z_t, g_t, \mu_t\}_{t=0}^\infty$ .

Households' optimality conditions are characterized as follows.

$$C_t + A_t = w_t L_t + (1 + r_t^a) A_{t-1}, \quad t \geq 0, \quad (\text{B.1})$$

$$(C_t - \kappa_R X_{t-1} L_t^{1+\omega})^{-\gamma} = \lambda_t, \quad t \geq 0, \quad (\text{B.2})$$

$$\lambda_t = \beta_R (1 + r_{t+1}^a) \lambda_{t+1}, \quad t \geq 0, \quad \text{and} \quad (\text{B.3})$$

$$w_t = (1 + \omega) \kappa_R X_{t-1} L_t^\omega, \quad t \geq 0. \quad (\text{B.4})$$

In this model, households do not save in deposits, and thus, equation (15) becomes

$$F_t = D_t, \quad t \geq 0. \quad (\text{B.5})$$

Firms' optimality conditions are the same as those in Online Appendix A.1, *i.e.*, equations (A.11) - (A.16). Asset return  $r_t^a$  and price  $q_t$  again satisfy equations (12) and (13). The interest rate  $r_t$  is determined by equation (18). Trade balance is determined by equation (20).

Given the initial conditions on  $X_{-1}, A_{-1}, K_{-1}, D_{-1}, F_{-1}$ , and  $r_{-1}$  and deterministic paths of aggregate shocks  $\{z_t, g_t, \mu_t\}_{t=0}^\infty$ , the equilibrium is characterized by variables  $\{C_t, A_t, L_t, \lambda_t, K_t, D_t, F_t, w_t, Y_t, I_t, \Pi_t, TB_t, q_t, r_t^a, r_t\}_{t=0}^\infty$  satisfying equations (B.1) - (B.5), (A.11) - (A.16), (12), (13), (18), and (20).

By Walras' law, the resource constraint (A.17) also holds in the RASOE economy.

This decentralized RASOE model is equivalent to the following centralized RASOE model, which appears far more frequently in related studies. Consider an economy composed of representative households who produce output  $Y_t$  using capital  $K_{t-1}$  and labor  $L_t$ , make investment  $I_t$  to accumulate capital, and borrow debt  $D_t$  from the international financial market. They optimize the GHH preference subject to resource constraints and the no-Ponzi-game constraint as follows.

$$\begin{aligned} \max_{\{C_t, I_t, K_t, Y_t, D_t, L_t\}_{t=0}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} (\beta_R)^t \frac{(C_t - \kappa_R X_{t-1} L_t^{1+\omega})^{1-\gamma}}{1-\gamma} \\ \text{s.t.} \quad & C_t + I_t + \frac{\phi}{2} \left( \frac{K_t}{K_{t-1}} - g^* \right)^2 K_{t-1} = Y_t + D_t - (1 + r_{t-1}) D_{t-1}, \\ & Y_t = z_t K_{t-1}^\alpha (X_t L_t)^{1-\alpha}, \\ & I_t = K_t - (1 - \delta) K_{t-1}, \text{ and} \\ & \lim_{j \rightarrow \infty} E_t \left[ D_{t+j} / \left( \prod_{s=0}^{j-1} (1 + r_{t+s}) \right) \right] \leq 0. \end{aligned}$$

Interest rate  $r_t$  in the international financial market and trade balance  $TB_t$  are determined according to equations (18) and (20), respectively.

It is straightforward to verify that i) all the equilibrium conditions in the centralized economy are satisfied in the decentralized economy and that ii)  $\{A_t, F_t, w_t, \Pi_t, q_t, r_t^a\}_{t=0}^{\infty}$  can be constructed in the centralized economy such that the equilibrium variables and the newly constructed variables of the centralized economy together satisfy all the equilibrium conditions of the decentralized economy. Therefore, the decentralized and centralized economies yield the same equilibrium.

## B.2 Detrended Equilibrium under Deterministic Paths of Aggregate Shocks

The equilibrium variables of the RASOE model are detrended as follows.

$$\begin{aligned} \tilde{x}_t &:= x_t / X_{t-1}, \quad x_t = C_t, w_t, Y_t, I_t, \Pi_t, TB_t, \\ \tilde{x}_t &:= x_t / X_t, \quad x_t = A_t, K_t, D_t, F_t, q_t, \quad \text{and} \\ \tilde{x}_t &:= x_t / X_{t-1}^{-\gamma}, \quad x_t = \lambda_t, \end{aligned}$$

Households' detrended optimality conditions are characterized as follows.

$$\tilde{C}_t + g_t \tilde{A}_t = \tilde{w}_t L_t + (1 + r_t^a) \tilde{A}_{t-1}, \quad t \geq 0, \quad (\text{B.6})$$

$$(\tilde{C}_t - \kappa_R L_t^{1+\omega})^{-\gamma} = \tilde{\lambda}_t, \quad t \geq 0, \quad (\text{B.7})$$

$$\tilde{\lambda}_t = g_t^{-\gamma} \beta_R (1 + r_t) \tilde{\lambda}_{t+1}, \quad t \geq 0, \quad \text{and} \quad (\text{B.8})$$

$$\tilde{w}_t = (1 + \omega) \kappa_R L_t^\omega, \quad t \geq 0. \quad (\text{B.9})$$

Equation (B.5) is detrended as follows.

$$\tilde{F}_t = \tilde{D}_t, \quad t \geq 0, \quad \text{and} \quad (\text{B.10})$$

Given the initial conditions on  $\tilde{A}_{-1}, \tilde{K}_{-1}, \tilde{D}_{-1}, \tilde{F}_{-1}$ , and  $r_{-1}$  and deterministic paths of aggregate

shocks  $\{z_t, g_t, \mu_t\}_{t=0}^\infty$ , the detrended equilibrium is characterized by variables  $\{\tilde{C}_t, \tilde{A}_t, L_t, \tilde{\lambda}_t, \tilde{K}_t, \tilde{D}_t, \tilde{F}_t, \tilde{w}_t, \tilde{Y}_t, \tilde{I}_t, \tilde{\Pi}_t, \tilde{T}B_t, \tilde{q}_t, r_t^a, r_t\}_{t=0}^\infty$  satisfying equations (B.6) - (B.10), (A.29) - (A.34), (A.40), (A.41), (A.44), and (A.45).

The detrended resource constraint (A.46) also holds in the RASOE economy.

## C Solution Method

I solve the detrended equilibrium of the HASOE and RASOE models using Auclert et al. (2021)'s method. The first step is to solve the steady state. To compute the steady state of the (detrended) HASOE economy, I solve workers' (detrended and normalized) policy functions and stationary distribution in the steady state using equations (A.18) - (A.26). Auclert et al. (2021) develop a fast algorithm that extends Carroll (2006)'s method of endogenous gridpoints to a two-asset environment. I closely follow this algorithm to solve workers' problem in the steady state. See Appendix E.1 of Auclert et al. (2021) for details of this algorithm.<sup>1</sup> The steady state of the (detrended) RASOE economy is straightforward to compute.

Once the steady state is pinned down, Auclert et al. (2021)'s method computes the Jacobians of 'blocks' along the directed acyclical graph (DAG). Here, a 'block' is a function that maps the sequences of input variables  $\{x_{1,t}, \dots, x_{n_x,t}\}_{t=0}^\infty$  into the sequences of output variables  $\{y_{1,t}, \dots, y_{n_y,t}\}_{t=0}^\infty$  using a subset of equilibrium conditions. The Jacobian of each block is a matrix composed of  $(\partial y_{j,s} / \partial x_{i,t})_{1 \leq i \leq n_x, 1 \leq j \leq n_y, s, t \geq 0}$ . For example, the worker block in my HASOE model maps the sequences of input variables  $\{\tilde{w}_t, r_t^a, r_t^b, g_t, \tilde{l}_t, \eta_t\}_{t=0}^\infty$  into the sequences of output variables  $\{\tilde{C}_t^W, \tilde{B}_t^W, \tilde{A}_t^W, \tilde{\chi}_t^W\}_{t=0}^\infty$  using equilibrium conditions (A.18) - (A.26) and (A.35) - (A.38).<sup>2</sup> The Jacobian of the worker block is composed of  $(\partial y_s / \partial x_t)_{x \in \{\tilde{w}, r^a, r^b, g, \tilde{l}, \eta\}, y \in \{\tilde{C}^W, \tilde{B}^W, \tilde{A}^W, \tilde{\chi}^W\}, s, t \geq 0}$ . Figures C.1 and C.2 present the DAG representation of the detrended HASOE and RASOE equilibria, respectively. By combining the Jacobians of the blocks along the DAG through the Chain Rule, Auclert et al. (2021)'s method solves the linearized dynamics of the equilibrium variables.

In the whole procedure of solving the HASOE model, i) computing the steady state and ii) computing the Jacobian of the heterogeneous worker block are the most time-consuming steps. In particular, calibrating  $\beta$ ,  $\chi_1$ , and  $\chi_2$  requires solving the steady state multiple times, and this step takes longer than a day. However, once these parameters are calibrated and both the computed steady state and Jacobian of the worker block are ready, the rest of the steps required to solve the model are very fast. This is why Bayesian estimation of the model is possible as long as the parameters to be estimated do not affect the steady state and the Jacobian of the worker block.

<sup>1</sup>For grids, I use 10 gridpoints for  $\hat{e}_1$  and  $\hat{e}_2$ , respectively, 50 gridpoints for  $\hat{b}_-$ , and 70 gridpoints for  $\hat{a}_-$ .

<sup>2</sup>Since I use normalized optimality conditions (A.18) - (A.25) to solve workers' problem, I first compute the Jacobian for normalized variables and then scale it to the one for unnormalized variables. For example, to compute  $(\partial \tilde{C}_s^W / \partial \tilde{w}_t)$ , I first compute  $(\partial \hat{C}_s^W / \partial \hat{w}_t)$ , and then scale it using the following equation:  $\frac{\partial \tilde{C}_s^W}{\partial \tilde{w}_t} = \frac{\partial (\tau(\bar{\Gamma}) \hat{C}_s^W)}{\partial (\tau(\bar{\Gamma}) \hat{w}_t)} = \frac{\tau(\bar{\Gamma})}{\tilde{w}_{ss}} \frac{\partial \hat{C}_s^W}{\partial \hat{w}_{ss}}$ .

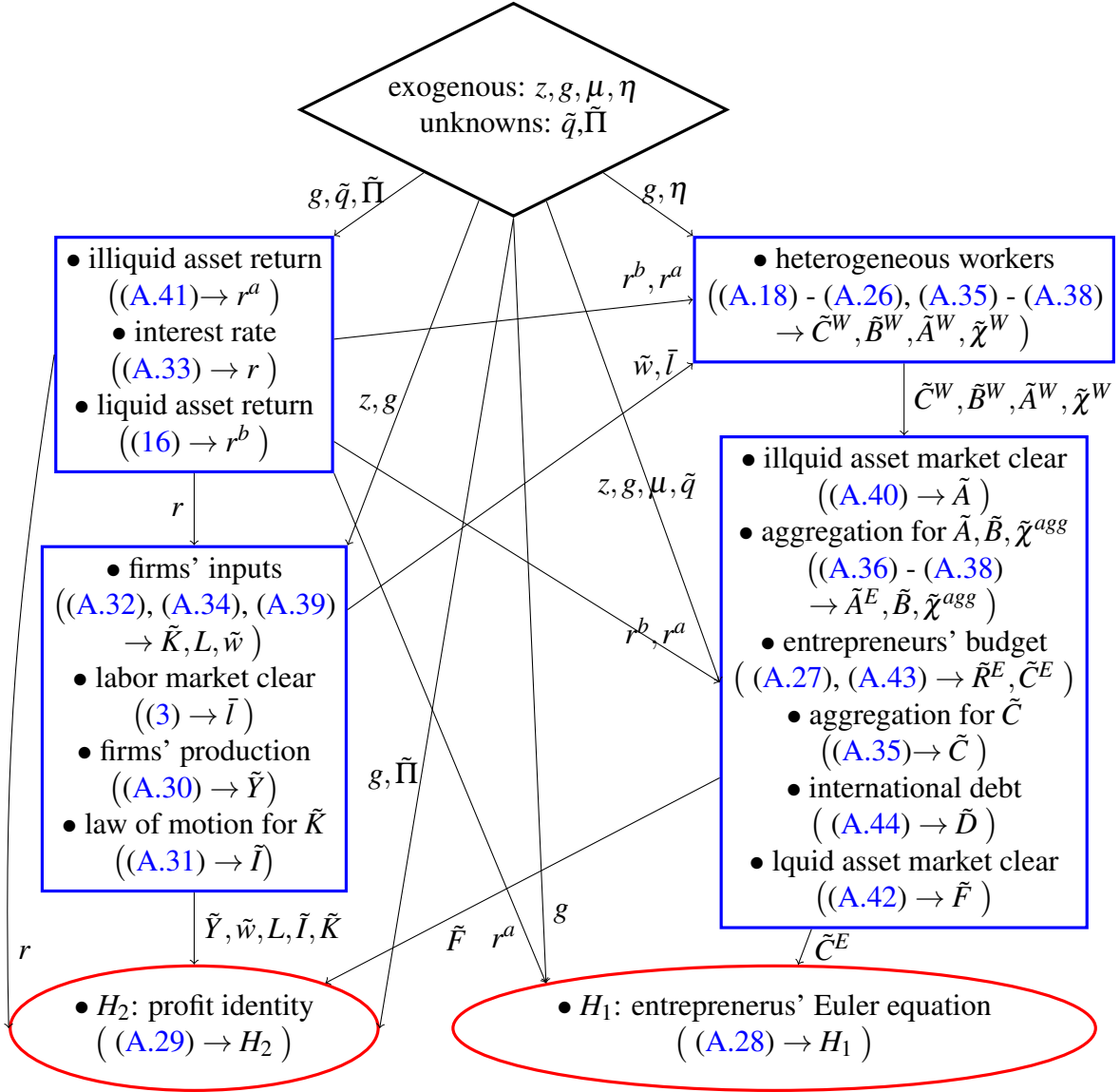


Figure C.1: DAG Representation of the Detrended Equilibrium of the HASOE Model

## D Truncation Errors

In implementation of Auclert et al. (2021)'s sequence space approach, the sequences of the equilibrium variables over an infinite horizon must be truncated to a finite horizon, and truncation errors can be nontrivial when the economy is extremely persistent. In this section, I inspect truncation errors and verify that at the posterior mode, the truncation errors are negligible in the model statistics used in this paper.

In this paper, I truncate sequences at  $T = 700$  when solving models and drop the last seven periods further when evaluating moments. I start by comparing the impulse response sequences (at the posterior mode) obtained under truncation at  $T = 700$  with those under truncation at  $T = 1500$ .

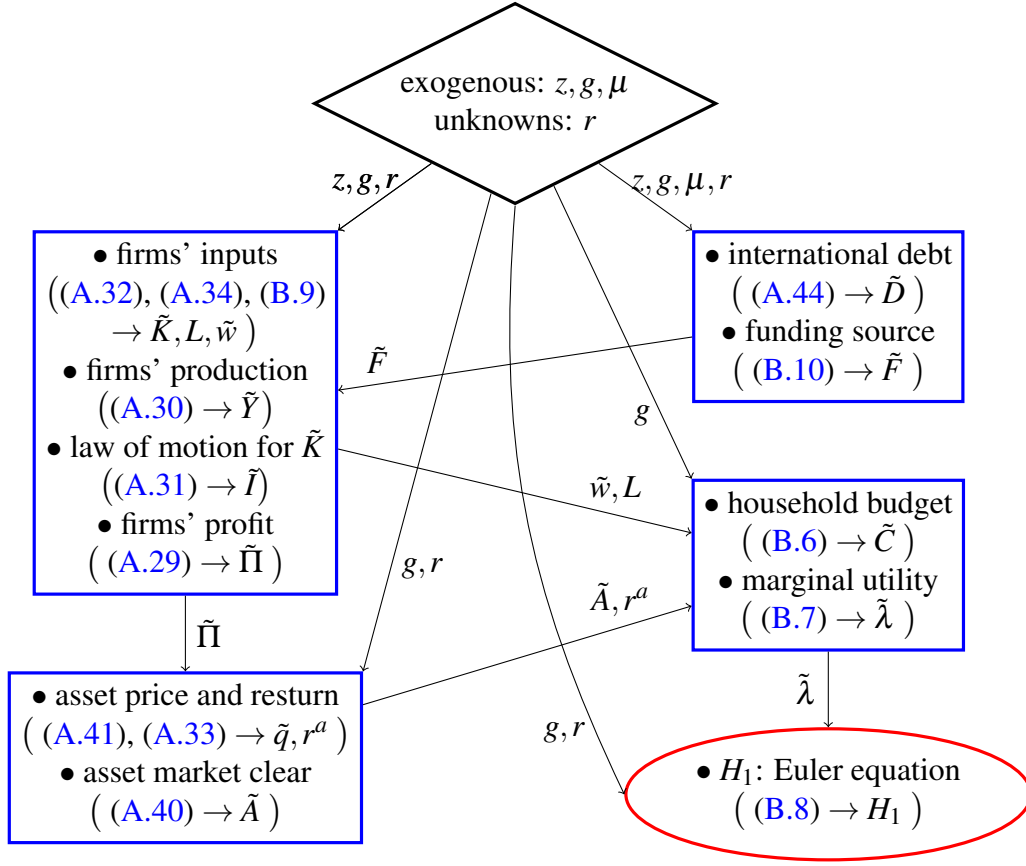


Figure C.2: DAG Representation of the Detrended Equilibrium of the RASOE Model

Three observations emerge from this comparison: i) in each model (*i.e.*, each of RASOE ( $z, g, \mu$ ), HASOE ( $z, g, \mu$ ), and HASOE ( $z, g, \mu, \eta$ ) models), all impulse responses reach a balanced growth path before  $t = 1499$ ; ii) there are some impulse responses that have not reached a balanced growth path in  $t = 699$ ; iii) even in such cases, the impulse response sequences obtained under truncation at  $T = 700$  are distorted only in the last few periods compared to those under truncation at  $T = 1500$ .

As an example, in Figure D.1, I plot the impulse responses of  $Y_t$ ,  $C_t$ ,  $I_t$ , and  $TB_t/Y_t$  to a trend shock in the RASOE model evaluated at its posterior mode, where the trend shock is very persistent

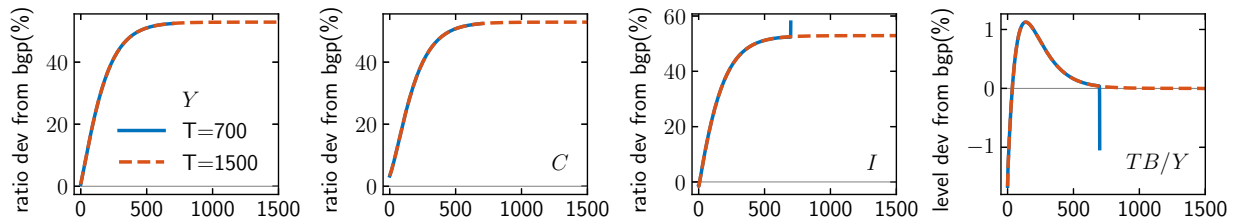


Figure D.1: Truncation Effects on the Impulse Responses to a  $g$  Shock in RASOE ( $z, g, \mu$ )

Notes: This figure plots the impulse responses of  $Y$ ,  $C$ ,  $I$ , and  $TB/Y$  to a one-standard-deviation  $g$  shock in the RASOE ( $z, g, \mu$ ) model at the posterior mode. The model is solved under two different truncation lengths,  $T = 700, 1500$ .



( $\rho_g = 0.988$ ). All the impulse responses in this figure reach a new balanced growth path before  $t = 1499$  (observation i)). The impulse responses of  $Y$  and  $C$  have already reached a balanced growth path before  $t = 699$ , while those of  $I$  and  $TB/Y$  have not (observation ii)). By truncating sequences at  $T = 700$ , a truncation error occurs in the impulse responses of  $I$  and  $TB/Y$ . Importantly, however, the truncation error occurs only in the last few periods (observation iii)).

Based on these observations, when I evaluate model moments, I drop the last  $0.01T$  periods from the length- $T$  sequences and use only the first  $0.99T$  periods. To examine truncation errors under this implementation, I evaluate the model statistics used in this paper at the posterior mode of each model under two different truncation lengths,  $T = 700$  and  $1500$ , and compare them. Specifically, I compare the following model statistics: i) the autocovarinaces of observable time series  $[\Delta \log Y_t, \Delta \log C_t, \Delta \log I_t, \Delta TB_t/Y_t]$ , which are used when evaluating a likelihood in the Bayesian estimation; ii) business cycle moments reported in Table 5; and iii) variance decomposition reported in Table 6. I find that truncation errors on these model statistics are negligible. As an example, in Figure D.2, I present the autocovarinaces of  $[\Delta \log Y_t, \Delta \log C_t, \Delta \log I_t, \Delta TB_t/Y_t]$  in the RASOE model under  $T = 700$  and  $T = 1500$ . (The other results are readily available upon request.)

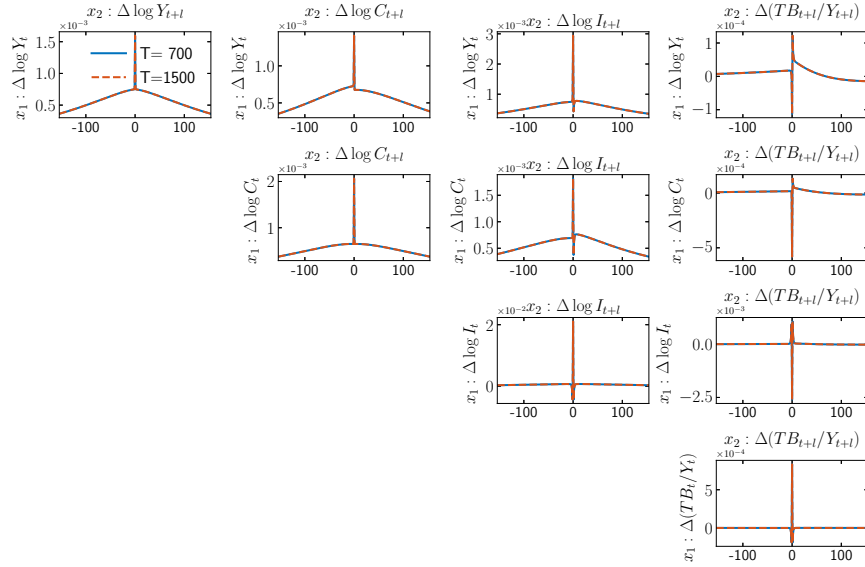


Figure D.2: Truncation Effects on the Autocovariances of Observables: RASOE ( $z, g, \mu$ ) Model

Notes: The RASOE ( $z, g, \mu$ ) model is evaluated at the posterior mode and solved at two different truncation lengths,  $T = 700$  and  $1500$ . The moments are evaluated using the first  $0.99T$  periods of the length- $T$  sequences.

## E MPC Estimation using Micro Data

### E.1 Method

As in [Hong \(2023\)](#), I estimate MPC out of transitory income shocks using an extended version of [Blundell et al. \(2008\)](#). Let the individual earnings  $Y_{i,t}$  be specified as follows.

$$\log Y_{i,t} = Z'_{i,t} \phi_t + P_{i,t} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim iid(0, \sigma_{tr}^2),$$

$$P_{i,t} = \rho P_{i,t-1} + \zeta_{i,t}, \quad \zeta_{i,t} \sim iid(0, \sigma_{ps}^2), \quad \text{and} \quad (\zeta_{i,t})_t \perp (\varepsilon_{i,t})_t.$$

where  $(x_t)_t$  represents time series  $(\dots, x_{t-1}, x_t, x_{t+1}, \dots)$ .  $Z'_{i,t} \phi_t$  represents the predictable component of log earnings  $\log Y_{i,t}$ , where  $Z_{i,t}$  denotes a vector of dummy variables for observable characteristics of household  $i$ .<sup>3</sup> The unpredictable component of log earnings,  $y_{i,t} (:= \log Y_{i,t} - Z'_{i,t} \phi_t)$ , is composed of a persistent component  $P_{i,t}$ , which follows an AR(1) process, and a transitory component, which is an *i.i.d.* shock. This earnings process specification is consistent with the HASOE model such that  $Z'_{i,t} \phi_t$ ,  $P_{i,t}$ , and  $\varepsilon_{i,t}$  correspond to  $\log(w_t \Gamma_i \bar{l}_t)$ ,  $\log e_{1,i,t}$ , and  $\log e_{2,i,t}$ , respectively.

Let  $c_{i,t}$  be the unpredictable component of consumption (*i.e.*,  $c_{i,t} := \log C_{i,t} - Z'_{i,t} \phi_t^c$ , where  $C_{i,t}$  is consumption and  $Z'_{i,t} \phi_t^c$  is the predictable component of  $\log C_{i,t}$ ). [Blundell et al. \(2008\)](#)'s partial insurance parameter to transitory shocks for a group  $G$ , which I denote by  $\psi_G$ , is defined as follows.

$$\psi_G = \frac{\text{cov}[\Delta c_{i,t}, \varepsilon_{i,t} | (i,t) \in G]}{\text{cov}[\Delta y_{i,t}, \varepsilon_{i,t} | (i,t) \in G]}.$$

In other words,  $\psi_G$  is the elasticity of consumption with respect to earnings when the earnings change is caused by an idiosyncratic transitory shock.

As in Online Appendix G.6 of [Hong \(2023\)](#), I estimate  $\psi_G$  adopting [Kaplan and Violante \(2010\)](#)'s identification strategy for [Blundell et al. \(2008\)](#)'s partial insurance parameters under the 'AR(1)+*i.i.d.*' specification of the earnings process. Specifically, let  $\tilde{\Delta}^K y_{i,t}$  and  $\Delta^K c_{i,t}$  be

$$\tilde{\Delta}^K y_{i,t} := y_{i,t} - \rho^K y_{i,t-K}, \quad \text{and} \quad \Delta^K c_{i,t} := c_{i,t} - c_{i,t-K}, \quad K \geq 1.$$

When the grouping of observations is independent of  $(\zeta_{i,t+j}, \varepsilon_{i,t+j})_{j \geq 0}$  and  $\Delta c_{i,t}$  is independent of  $(\zeta_{i,t+j}, \varepsilon_{i,t+j})_{j \geq 1}$ , we can derive

$$\psi_G = \frac{\text{cov}[\Delta^K c_{i,t}, \tilde{\Delta}^K y_{i,t+K} | (i,t) \in G]}{\text{cov}[\tilde{\Delta}^K y_{i,t}, \tilde{\Delta}^K y_{i,t+K} | (i,t) \in G]}. \quad (\text{E.1})$$

To identify  $\psi_G$  using equation (E.1), we need the value of  $\rho$ . Adopting [Floden and Lindé \(2001\)](#)'s identification strategy, parameter  $\rho$  is estimated using the following moment conditions.<sup>4</sup>

$$E[y_{i,t}^2] = \frac{\sigma_{ps}^2}{1 - \rho^2} + \sigma_{tr}^2, \quad \text{and} \quad E[y_{i,t}, y_{i,t+nK}] = \frac{\sigma_{ps}^2}{1 - \rho^2} \rho^{nK}, \quad n \geq 1. \quad (\text{E.2})$$

Once  $\rho$  is estimated, I estimate  $\psi_G$  using equation (E.1). Since  $\psi_G$  is an elasticity, I transform it to MPC by multiplying a group-level consumption-income ratio as follows.

$$MPC_G = \psi_G \frac{E[C_{i,t} | (i,t) \in G]}{E[Y_{i,t} | (i,t) \in G]}. \quad (\text{E.3})$$

<sup>3</sup>The observable characteristics of households include education, ethnicity, employment status, region, cohort, household size, number of children, urban area, the existence of members other than heads and spouses earning income, and the existence of persons who do not live with but are financially supported by the household. Among these characteristics, education, ethnicity, employment status, and region are allowed to have time-varying effects.

<sup>4</sup>In this estimation, I obtain the estimates of  $\rho$ ,  $\sigma_{ps}$ , and  $\sigma_{tr}$ . These estimates for Peruvian households are also used in subsection 3.1 to calibrate the earnings process in the HASOE model.

ENAH0 provides the year-over-year growth of quarterly income and consumption, and thus, I set one period as a quarter and  $K = 4$  for the Peruvian sample. As a result, I obtain quarterly MPCs of Peruvian households. On the other hand, the PSID provides the two-year-over-two-year growth of annual income and consumption, and thus, I set one period as a year and  $K = 2$  for the U.S. sample. As a result, I obtain annual MPCs of U.S. households.

## E.2 Revisions on [Hong \(2023\)](#)

Compared to [Hong \(2023\)](#), I make three revisions to the MPC estimation procedure, which are necessary to maintain consistency between micro moments and macro data or the model. First, I change the consumption measure from non-durable consumption to total consumption (including both non-durable and durable consumption). Once the model is calibrated by targeting the MPC moments, I estimate the model using macro data. In this step, I use total consumption series (as studies on emerging market business cycles typically do) because non-durable consumption is not available in the Peruvian national accounts. To make the consumption concept consistent between micro and macro data, I use the total consumption measure when analyzing the micro data, too.

Second, the sample periods are changed for both ENAH0 and the PSID because some of the key durable expenses are available only after certain years in both surveys. Specifically, I use the 2011-2018 waves of ENAH0 and the 2005-2017 waves of the PSID.<sup>5</sup>

Third, the earnings process specification is revised. In its baseline specification, [Hong \(2023\)](#) assumes that  $P_{i,t}$  follows a random walk as in the original specification of [Blundell et al. \(2008\)](#). In this paper, I instead assume that  $P_{i,t}$  follows an AR(1) process so that the earnings process specification imposed in the MPC estimation is consistent with the model.<sup>6</sup>

## E.3 Variable Construction

The consumption measure in MPC estimation is total consumption, which includes both non-durable and durable consumption. I construct such consumption by aggregating the following

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<sup>5</sup>My ENAH0 sample starts from 2011 for the following reason. ENAH0 is conducted continuously (*i.e.*, households are interviewed in different months) and the reference periods of income and expense items are usually in the format of a ‘specified period before the interview’ (such as ‘previous  $n$  months’) rather than a fixed calendar period (such as ‘during 2014’). Accordingly, I set the reference periods of my consumption and income measures using the same format (*i.e.*, a specified period before the interview such as ‘previous  $n$  months’). One exception is Questionnaire 612. This questionnaire collects information on household furnishings, equipment, and vehicles, which take a sizable portion of durable goods. Until 2010, this questionnaire asks which calendar year each item is acquired, and thus it is impossible to aggregate this questionnaire’s expense items with other expense items under a consistent reference period format. From 2011 onward, Questionnaire 612 asks the acquisition month instead of the acquisition year, which makes it possible to recover this questionnaire’s expense items during a specified period before the interview (such as ‘previous  $n$  months’) and to aggregate these expense items with other expense items under a consistent reference period format. My PSID sample starts from 2005 because the survey began to collect expenses on household furnishings and equipment since then. Moreover, some non-durable items including clothing and recreation are also collected from 2005 onward.

<sup>6</sup>[Hong \(2023\)](#) also considers this ‘AR(1) + *i.i.d.*’ specification during a robustness check in his Appendix G.6.

expenses in each of ENAHO and the PSID: non-durable expenses including 1) food, 2) clothing (including clothing services, footwear, watches and jewelry), 3) housing rent, rental equivalence of owned or donated housing, 4) utilities (heat, electricity, water, etc.), 5) telephone and cable, 6) vehicle repairs and maintenance, 7) gasoline and oil, 8) parking, 9) public transportation, 10) household repairs and maintenance, 11) recreation, 12) insurance (home insurance, car insurance, health insurance, etc.), 13) childcare, 14) domestic services and other home services, 15) personal care, 16) alcohol, 17) tobacco, and 18) daily non-durables (laundry items, bathroom items, matches, candle, stationeries, etc.), and durable expenses including 19) vehicles, 20) furnishings and equipment (textiles, furniture, floor coverings, appliances, housewares, etc.), 21) health, and 22) education.<sup>7</sup> Among the listed expenses, ENAHO does not have expenses on 13) childcare, and the PSID does not have expenses on 14) domestic services and other home services, 15) personal care, 16) alcohol, 17) tobacco, and 18) daily non-durables (laundry items, bathroom items, matches, candle, stationeries, etc.). Nonpurchased consumption, such as donations, food stamps, in-kind income, and self-production, is excluded.

The income measure in MPC estimation is the sum of disposable labor income and transfers, as in [Blundell et al. \(2008\)](#). Capital income is excluded in order not to falsely attribute endogenous capital income changes as income shocks. In ENAHO, capital income and labor income are not distinguishable in self-employment income. As in [Diaz-Gimenez et al. \(1997\)](#), [Krueger and Perri \(2006\)](#), and [Hong \(2023\)](#), I split the self-employment income into labor income and capital income parts using the ratio between unambiguous capital and labor incomes in the sample.<sup>8</sup> In ENAHO, imputed components of missing income are distinguishable, and I exclude them from Peruvian incomes, as in [Hong \(2023\)](#). For the PSID sample, I closely follow [Kaplan et al. \(2014\)](#) in constructing U.S. incomes. Specifically, U.S. households' disposable labor income and transfers are constructed by i) estimating federal income taxes for total income (including labor income, transfers, and capital income) by TAXSIM program, ii) splitting proportionately the estimated federal taxes into the labor income and transfers part and the capital income part, and iii) subtracting the federal taxes on labor income and transfers from gross labor income and transfers.

In ENAHO, reference periods vary across income and expense items. Importantly, Peruvian households report 97.5% of income items and 92.9% of expense items (in value) under reference periods shorter than or equal to the previous three months, on average. Given this feature of the data, I set the reference period of Peruvian income and consumption as the previous three months.

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<sup>7</sup>In listing the expenses, I categorize expenses on 21) health and 22) education as durable expenses because of their durable nature. In national accounts, however, they are categorized as non-durable consumption. Since I use total consumption, how they are categorized between durable and non-durable consumption does not have any effect.

<sup>8</sup>As noted in footnote 23, the ratio of (unambiguous labor income) / (unambiguous labor income + unambiguous capital income) is 0.817 in ENAHO, and this ratio is close to the ratio that [Diaz-Gimenez et al. \(1997\)](#) and [Krueger and Perri \(2006\)](#) use for their U.S. sample, 0.864.

Expense and income items reported under a different reference period than the previous three months are scaled to three-month expenses and incomes, respectively. (For example, a monthly tobacco expense is scaled up by a factor of three.)<sup>9</sup> Moreover, to remove any comovement between income and consumption that occurs prior to the previous three months, I exclude income items with reference periods longer than the previous three months from Peruvian incomes.

In the PSID, the reference periods of income items are firmly fixed to a calendar year, while the reference periods of expense items can depend on interpretation, as [Crawley \(2020\)](#) notes. For example, food expenses in the PSID can be interpreted either as the last week's expense or the average weekly expense during the calendar year. I adopt the latter interpretation, as related studies often do, and treat the reference periods of expense items as being synchronized with those of income items. Accordingly, I set the reference period of U.S. income and consumption as the corresponding calendar year.

ENAH0 is conducted annually, and I use the 2011-2018 waves. This ENAH0 sample provides seven years of the year-over-year growth of quarterly income and consumption. For the PSID, I use the 2005-2017 waves, and it is conducted biannually during the sample period. This PSID sample provides six years of the two-year-over-two-year growth of annual income and consumption.

In both the ENAH0 and the PSID samples, nominal income and consumption are deflated with the Consumer Price Index (CPI) series.<sup>10</sup>

#### **E.4 Sample Selection**

The sample selection for ENAH0 closely follows [Hong \(2023\)](#) and proceeds as follows. First, I drop observations when households appear only once in the survey. Second, I drop observations when households are interviewed in different months between two consecutive surveys or when household heads are changed. I also drop observations when it is likely that two different households are linked as panel observations by failing to distinguish an old household moving out and a new household moving into the same address.<sup>11</sup> Third, I drop observations classified as 'incomplete' by pollsters. Fourth, I drop observations when household heads are younger than 25 or older than 65. Fifth, I drop observations when observable characteristics used to control for the predictable components of income and consumption are missing. Sixth, I drop observations reporting

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<sup>9</sup>Online Appendix B.2 of [Hong \(2023\)](#) describes how one can achieve such scaling effectively using certain variables in the ENAH0 data.

<sup>10</sup>Unlike the reference periods in the PSID sample, the reference periods in the ENAH0 sample are not fixed to a calendar period. For example, the three-month reference period of households surveyed in January, 2015 starts one-month earlier than that of households surveyed in February, 2015. Fortunately, this feature of the data does not complicate the deflation procedure, as ENAH0 provides variables recording within-year-deflated values of income and expense items. Online Appendix B.2 of [Hong \(2023\)](#) provides a detailed deflation procedure using these variables.

<sup>11</sup>[Hong \(2023\)](#) defines such panel observations as 'potentially fake panel observations.' The potentially fake panel observations can be effectively detected and dropped by using household-member-level information. See Online Appendix B.4 of [Hong \(2023\)](#) for a detailed discussion.

nonpositive income or consumption. Seventh, I drop observations with too much imputed value or too much value reported under a longer reference period than the previous three months in their income.<sup>12</sup> Eighth, I drop income outliers.<sup>13</sup> As a result, I obtain a sample composed of 36,292 observations, 18,479 pairs of two consecutive observations, and 7,241 triplets of three consecutive observations.

The sample selection for the PSID proceeds similarly to the sample selection for ENAHO as follows. First, I drop observations when households appear only once in the survey. I also drop observations when household heads are changed. Second, I drop observations if they belong to the sample from Survey of Economic Opportunities (SEO) (added to the PSID in 1968) or to the Latino sample (added to the PSID in 1990 and 1992). Third, I drop observations when their income or consumption include topcoded values. Fourth, I drop observations when household heads are younger than 25 or older than 65. Fifth, I drop observations when observable characteristics used to control for the predictable components of income and consumption are missing. Sixth, I drop observations reporting nonpositive income or consumption. Seventh, I drop income outliers, which are defined in the same way as in the Peruvian sample selection. As a result, I obtain a sample composed of 29,145 observations, 22,345 pairs of two consecutive observations, and 16,092 triplets of three consecutive observations.

## E.5 Earnings Grouping

When disciplining the HASOE model, I use MPC estimated within each decile of residual earnings, which is  $e_{i,t}$  in the model and  $y_{i,t}$  in the MPC estimation procedure described in Online Appendix E.1. In particular, I do not group observations by total earnings, which is  $w_t \Gamma_i e_{i,t} \bar{l}_t$  in the model and  $Y_{i,t}$  in the MPC estimation procedure, because  $e_{i,t}$  bears risk and thus induces precautionary saving and MPC heterogeneity, while  $\Gamma_i$  does not. Residual earnings deciles are constructed in the same way as in Hong (2023). See section 3.4 of the paper for details.

## F The Market Value of Wealth

As noted in footnote 26, when calibrating  $p$ , I evaluate the market value of entrepreneurs' claims to rents  $R_t^E$  assuming that they can trade the claims among themselves and include this value as part of their wealth. Specifically, the market value of the claims is evaluated as follows.

$$U_t^E := \sum_{s=1}^{\infty} (\beta^E)^s \frac{(C_{t+s}^E)^{-\gamma}}{(C_t^E)^{-\gamma}} R_{t+s}^E = \sum_{s=1}^{\infty} \frac{Q_{0,t+s}}{Q_{0,t}} \frac{\chi_{t+s}^{agg} + \xi(1 + r_{t+s}^b)B_{t+s-1}}{1 - p}.$$

<sup>12</sup>‘Too much value’ is defined as follows. For each  $(x, y) \in \{(\text{imputed value, baseline income}) (\text{value reported under a reference period longer than the previous three months, baseline income})\}$ , I drop observations when  $\frac{x}{x+y} > 0.05$ .

<sup>13</sup>Income outliers are defined as households exhibiting an extreme income growth, which falls in the range of extreme 1 % (0.5% at the top and 0.5% at the bottom) in a calendar-year subsample at least once.



In the steady state of the detrended equilibrium,  $\tilde{U}_t^E := U_t^E / X_t$  becomes

$$\tilde{U}_{ss}^E = \sum_{s=1}^{\infty} \frac{1}{(1+r_{ss})^s} \frac{\tilde{\chi}_{ss}^{agg} + \xi(1+r_{ss})\tilde{B}_{ss}}{1-p} (g_{ss})^{s-1} = \frac{\tilde{\chi}_{ss}^{agg} + \xi(1+r_{ss})\tilde{B}_{ss}}{1+r_{ss}-g_{ss}} \frac{1}{1-p},$$

and entrepreneurs' and workers' wealth,  $\tilde{\mathcal{W}}_{ss}^E$  and  $\tilde{\mathcal{W}}_{ss}^W$ , respectively, are characterized by

$$\tilde{\mathcal{W}}_{ss}^E = \tilde{U}_{ss}^E + \tilde{A}_{ss}^E, \quad \text{and} \quad \tilde{\mathcal{W}}_{ss}^W = \tilde{B}_{ss}^W + \tilde{A}_{ss}^W.$$

The market value of the total wealth in this economy,  $\tilde{\mathcal{W}}_{ss}$ , is characterized as follows.

$$\begin{aligned} \tilde{\mathcal{W}}_{ss} &= \underbrace{p\tilde{\mathcal{W}}_{ss}^W}_{\text{workers' portion}} + \underbrace{(1-p)\tilde{\mathcal{W}}_{ss}^E}_{\text{entrepreneurs' portion}} \\ &= \tilde{A}_{ss} + \tilde{B}_{ss} + (1-p)\tilde{U}_{ss}^E = \tilde{K}_{ss} - \tilde{D}_{ss} + (1-p)\tilde{U}_{ss}^E = \tilde{K}_{ss} - \tilde{D}_{ss} + \frac{\tilde{\chi}_{ss}^{agg} + \xi(1+r_{ss})\tilde{B}_{ss}}{1+r_{ss}-g_{ss}}. \end{aligned} \quad (\text{F.1})$$

Parameter  $p$  is calibrated such that the fraction  $(1-p)\tilde{\mathcal{W}}_{ss}^E/\tilde{\mathcal{W}}_{ss}$  in the model under a given value of  $p$  is equal to the top 100(1-p)% share of wealth in the data (WID).

## G Consumption Response Decomposition in the RASOE Model

In Figure 2a, I decompose the response of  $GHH_t$  to a trend shock ( $g$ ) in the RASOE model into the response driven by  $w_t$  and  $h_t(\cdot)$  and the response driven by  $r_t^a$ . This consumption response decomposition, however, cannot be obtained directly from the sequence space approach. This is because the DAG representation of the RASOE model (presented in Figure C.2) must be written such that households' partial equilibrium problem does not form a separate block for the following reason: if representative households' partial equilibrium problem forms a separate block, the Jacobian of the block contains  $(\partial GHH_s/r_t^a)_{s,t \geq 0}$ , which, for any  $t$ , never dies out when  $s \rightarrow \infty$ .

Thus, I obtain the consumption response decomposition in the RASOE model using the conventional state space approach developed by [Blanchard and Kahn \(1980\)](#). Let

$$dy_t = d\tilde{C}_t, \quad dx_t = [d\tilde{A}_{t-1}, dg_t, d\tilde{w}_t, dr_t^a]', \quad \text{and} \quad d\epsilon_t = [dg_t, d\tilde{w}_t, dr_t^a]'$$

The households' partial equilibrium is characterized by detrended equilibrium conditions (B.6) - (B.9). After substituting out  $L_t$  and  $\tilde{\lambda}_t$  from the equilibrium conditions, the households' partial equilibrium can be described by the following VAR representation.

$$A \cdot \begin{bmatrix} dy_{t+1} \\ dx_{t+1} \end{bmatrix} = B \cdot \begin{bmatrix} dy_t \\ dx_t \end{bmatrix} + C \cdot d\epsilon_{t+1}, \quad (\text{G.1})$$

where  $A$  and  $B$  are 5-by-5 matrices and  $C$  is a 5-by-3 matrix. Note that  $dy_t$  is a control variable,  $dx_t$  is a vector of state variables, and  $d\epsilon_t$  is a vector of exogenous variables in this system. I verify that under any posterior parameter draws,  $A$  is invertible, and  $A^{-1}B$  satisfies [Blanchard and Kahn \(1980\)](#)'s stationarity condition. By solving the system (G.1) with [Blanchard and Kahn](#)



(1980)’s method and feeding the impulse responses of  $d\epsilon_t (= [dg_t, d\tilde{w}_t, dr_t^a]')$  to a trend shock into the system, one can obtain the impulse response of  $dy_t (= d\tilde{C}_t)$ . The consumption response can be decomposed into the response driven by  $w_t$  and  $h_t(\cdot)$  and the response driven by  $r_t^a$  by substituting  $d\epsilon_t$  with  $[dg_t, d\tilde{w}_t, 0]'$  and  $[0, 0, dr_t^a]'$ , respectively.

## H Workers’ Consumption Response to Future Aggregate Earnings Growth under a Low-Risk or Low-Financial-Friction Environment

In this section, I examine how workers’ consumption ( $C^W$ ) response to future aggregate earnings ( $w\bar{l}$ ) growth changes in the HASOE model when workers face lower idiosyncratic risk or lower financial friction than the baseline economy. To this end, I scale down either  $\sigma_{\epsilon_1}$  and  $\sigma_{\epsilon_2}$  (idiosyncratic risk) or  $\chi_1$  (financial friction) by half.

Table H.1 reports how workers’ aggregate savings change. Under both low-idiosyncratic-risk and low-financial-friction environments, workers save significantly less than in the baseline economy. This is because under both environments, workers’ precautionary saving behavior becomes significantly weaker, as they concern less about a future low income path: in the former environment, the probability of a future low income path is lower; in the latter environment, the cost of liquidating assets is cheaper even under the realization of a low income path.<sup>14</sup>

Figure H.1 shows how workers’ consumption response to the same future aggregate earnings ( $w\bar{l}$ ) growth changes in the low-idiosyncratic-risk or low-financial-friction environment. The first panel plots a workers’ consumption ( $C^W$ ) response in the baseline HASOE model to the aggregate earnings ( $w\bar{l}$ ) path plotted in the fourth panel of Figure 2b. (As explained in subsection 3.4, the aggregate earnings path is driven by a trend shock in the HASOE model evaluated at the parameter draws from the RASOE model’s posterior distribution.) The graph in this first panel is identical to the blue graph in the third panel of Figure 2b after the relevant scaling for the different y-axes.<sup>15</sup>

Table H.1: Workers’ Savings under Low Idiosyncratic Risk or Low Financial Friction

	Savings( $A^W + B^W$ )/ $Y$	$A^W/Y$	$B^W/Y$
Baseline HASOE	6.658	6.016	0.642
Low idiosyncratic risk ( $0.5\sigma_{\epsilon_1}$ & $0.5\sigma_{\epsilon_2}$ )	1.223	1.057	0.166
Low financial friction ( $0.5\chi_1$ )	1.054	0.274	0.780

<sup>14</sup>Under the low-financial-friction environment, workers use liquid assets as the main saving vehicle rather than illiquid assets. Since the illiquid asset adjustment cost has  $\{(1 + r_t^a)a_{i,t-1} + \chi_0 \Upsilon(\Gamma_i)X_{i-1}\}^{\chi_2-1}$  in the denominator, the cost becomes extremely large as illiquid asset position approaches zero, despite a low value of  $\chi_1$ . Thus, in the low-financial-friction environment where workers’ illiquid asset position is on average close to zero, liquid assets can be a cheaper tool than illiquid assets for consumption smoothing, despite their low return.

<sup>15</sup>The y-axis is ‘the ratio deviation from the balanced growth path of  $C$ ’ in the third panel of Figure 2b while that of  $C^W$  in the first panel of Figure H.1. Total consumption  $C$  includes  $C^E$ , which is also affected (on the balanced growth path) via  $\chi^{agg}$  when idiosyncratic risk or financial friction are lowered. By using the y-axis of ‘the ratio deviation from the balanced growth path of  $C^W$ ’ (instead of that of  $C$ ), the graphs in Figure H.1 are insulated from the change of  $C^E$ .

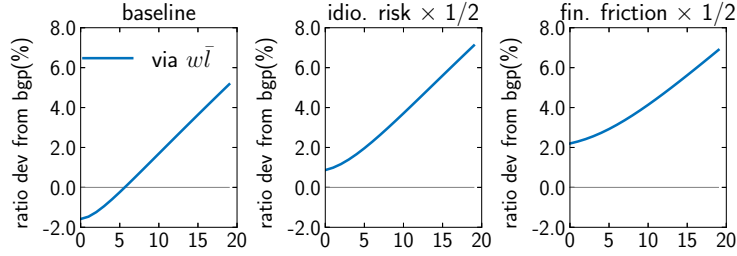


Figure H.1: Workers' Consumption ( $C^W$ ) Response to Future Aggregate Earnings ( $w\bar{l}$ ) Growth under Low Idiosyncratic Risk or Low Financial Friction

*Notes:* The first panel plots a workers' consumption ( $C^W$ ) response in the baseline HASOE model to the aggregate earnings ( $w\bar{l}$ ) path plotted in the fourth panel of Figure 2b. The second and third panels plot workers' consumption responses to the same aggregate earnings path but under a lower-idiosyncratic-risk environment where  $\sigma_{e_1}$  and  $\sigma_{e_2}$  are scaled down by half and a lower-financial-friction environment where  $\chi_1$  is scaled down by half, respectively.

The second panel plots workers' consumption response to the same aggregate earnings path but under lower idiosyncratic risk. The initial response of workers' consumption turns positive, reaching approximately 1% of its balanced growth path. The consumption responses in the subsequent periods are also greater than in the first panel and by a similar magnitude over time, resulting in a roughly the same slope of the impulse response graph. The parallel upward shift of the impulse response graph compared to that in the first panel is because of workers' weakened precautionary saving due to lower idiosyncratic risk. The slope does not flatten because workers still face strong financial friction ( $\chi_1$ ), and thus, it is still expensive to borrow from the future by cashing out assets.

The third panel plots workers' consumption response to the same aggregate earnings path but under lower financial friction. The initial response of workers' consumption is not only positive but also quite large, reaching above 2% of its balanced growth path. The consumption responses in the subsequent periods are also greater than in the first panel but by a decreasing magnitude over time, resulting in a less steep impulse response graph. The impulse response graph shifts upward compared to that in the first panel because i) lower financial friction makes it less expensive for workers to borrow from the future by cashing out their assets and ii) also weakens workers' precautionary saving behavior. The graph does not completely flatten because the other forms of financial friction than the asset adjustment cost, the hard borrowing constraints ( $a \geq 0$ ,  $b \geq 0$ ), become effective as savings become close to the borrowing limits.

## I A Microfoundation on the Illiquid Asset Adjustment Cost

In this section, I formalize the haircut interpretation that I discuss in section 4 by providing a microfoundation, closely following [Fostel and Geanakoplos \(2015\)](#), in which the illiquid asset adjustment cost is the haircut of collateralized borrowing. This microfoundation provides two important takeaways for the model. First, both the borrowers' default risk and the market liquidity risk

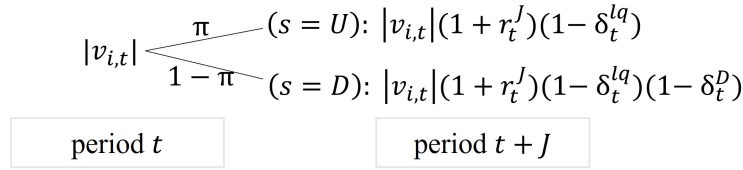


Figure I.1: Two States on the Value of the Collateral for a  $J$ -period Loan

are important determinants of the haircut size (or, equivalently, the illiquid asset adjustment cost in the model). Second, the part of the haircut rate due to borrowers' default risk can be recovered from the spread between the finance rate of the borrowing and the asset return rate.

In each period  $t$ , households can borrow money for  $J$  periods by collateralizing a part of their illiquid assets. Let  $|v_{it}|$  be the amount of collateralized assets. The expected value of the collateral in period  $t+J$  is affected by three factors (see Figure I.1): i) the illiquid asset return over the  $J$  periods,  $1+r_t^J := \prod_{s=1}^J (1+r_{t+s}^a)$ , ii) an expected discount rate  $\delta_t^{lq}$  for the liquidation at maturity due to the market liquidity condition, and iii) a borrower-specific idiosyncratic shock of size  $\delta_t^D$  to the collateral value at maturity, which is realized only with probability  $1-\pi$ . Note that the first two factors (asset return and market liquidity) bear aggregate risk, while the last factor (borrower-specific loss) bears idiosyncratic risk. For notational convenience, let  $D$  and  $U$  denote the state with and without the realization of the idiosyncratic loss, respectively.

A debt contract is characterized by the size of collateral  $|v_{i,t}|$  and the promised payment  $j$  at maturity  $t+J$ . At maturity, borrowers pay back the promised payment  $j$  only when the collateral value is greater than the promised payment; otherwise, they default. Therefore, the actual debt payoff is determined as follows (see Figure I.2a): under state  $U$ , the debt payoff equals  $j$  when  $j < |v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})$  because borrowers pay back the promised payment, while the payoff equals  $|v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})$  when  $j \geq |v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})$  because borrowers default and lenders collect the collateral value; under state  $D$ , the cutoff on  $j$  for borrowers' default shifts from  $|v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})$  to  $|v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})(1-\delta_t^D)$ .

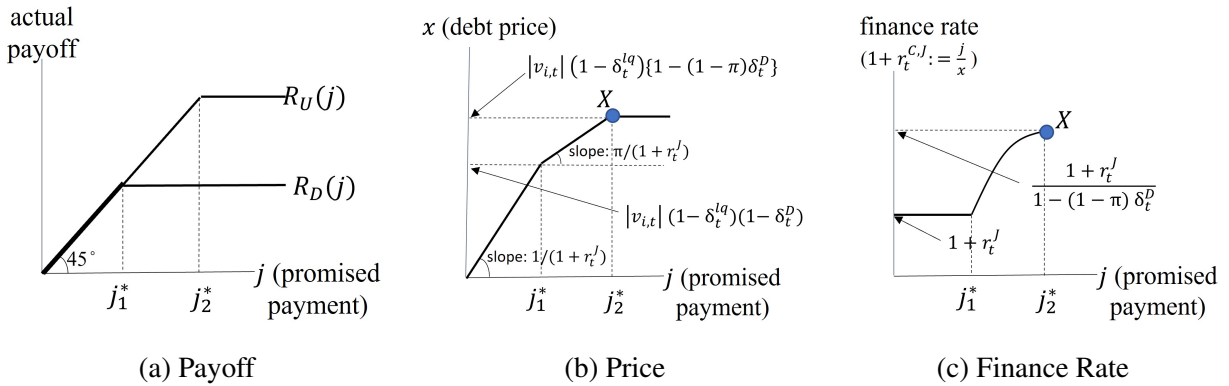


Figure I.2: Payoff, Price, and Finance Rate of the Collateralized Loan

Notes: In each figure,  $j_1^* = |v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})(1-\delta_t^D)$  and  $j_2^* = |v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})$ .

Under the assumption that lenders are risk-neutral and exhibit the same discount factor as firms,  $Q_{t,t+J} = 1/(1+r_t^J)$ , the debt price  $x_{it}$  is determined as follows.

$$\begin{aligned} x_{it} &= \frac{j}{1+r_t^J} & \text{if } j < j_1^* &:= |v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})(1-\delta_t^D), \\ x_{it} &= \frac{\pi j + (1-\pi)|v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})(1-\delta_t^D)}{1+r_t^J} & \text{if } j_1^* \leq j < j_2^* &:= |v_{i,t}|(1+r_t^J)(1-\delta_t^{lq}), \text{ and} \\ x_{it} &= |v_{i,t}|(1-\delta_t^{lq})\{1-(1-\pi)\delta_t^D\} & \text{if } j \geq j_2^*. \end{aligned} \quad (\text{I.1})$$

Figure I.2b illustrates the debt price graph over the axis of  $j$ . Lenders do not accept  $j > |v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})$  because borrowers will never be able to pay the promised payment in any state. Within the acceptable range,  $j \leq |v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})$ , households want to maximize the borrowing amount  $x_{it}$  given the size of collateral  $|v_{i,t}|$  and thus choose point X in Figure I.2b. At point X, the haircut rate  $\chi_t(v_{i,t})/|v_{i,t}|$  is determined as follows.

$$\frac{\chi_t(v_{i,t})}{|v_{i,t}|} = \frac{|v_{i,t}| - x_{it}}{|v_{i,t}|} = 1 - (1-\delta_t^{lq})\{1-(1-\pi)\delta_t^D\}. \quad (\text{I.2})$$

Given the promised payoff  $j$  and the debt price  $x_{it}$ , the finance rate of this loan over  $J$  periods,  $r_t^{C,J}$ , is determined by  $1+r_t^{C,J} = (j/x_{it})$ . Using equations (I.1), we can characterize  $r_t^{C,J}$  as follows.

$$\begin{aligned} 1+r_t^{C,J} &= 1+r_t^J & \text{if } j < j_1^* &:= |v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})(1-\delta_t^D), \\ 1+r_t^{C,J} &= \frac{(1+r_t^J)j}{\pi j + (1-\pi)|v_{i,t}|(1+r_t^J)(1-\delta_t^{lq})(1-\delta_t^D)} & \text{if } j_1^* \leq j < j_2^* &:= |v_{i,t}|(1+r_t^J)(1-\delta_t^{lq}), \text{ and} \\ 1+r_t^{C,J} &= \frac{j}{|v_{i,t}|(1-\delta_t^{lq})\{1-(1-\pi)\delta_t^D\}} & \text{if } j \geq j_2^*. \end{aligned} \quad (\text{I.3})$$

Figure I.2c illustrates the graph of the finance rate  $1+r_t^{C,J}$  over the axis of  $j$ . At point X, the finance rate is determined as follows.

$$1+r_t^{C,J} = \frac{1+r_t^J}{1-(1-\pi)\delta_t^D}. \quad (\text{I.4})$$

From equation (I.4), we have ‘ $1-(1-\pi)\delta_t^D = (1+r_t^J)/(1+r_t^{C,J})$ .’ By substituting this equation into equation (I.2), we can obtain

$$\frac{\chi_t(v_{i,t})}{|v_{i,t}|} = 1 - \frac{(1-\delta_t^{lq})(1+r_t^J)}{1+r_t^{C,J}} = \frac{(r_t^{C,J} - r_t^J) + \delta_t^{lq} + r_t^J \delta_t^{lq}}{1+r_t^{C,J}}.$$

By ignoring the second-order term  $r_t^J \delta_t^{lq}$ , the haircut rate can be rewritten as

$$\underbrace{\frac{\chi_t(v_{i,t})}{|v_{i,t}|}}_{\text{haircut rate}} \approx \underbrace{\frac{r_t^{C,J} - r_t^J}{1+r_t^{C,J}}}_{\text{borrowers' default risk}} + \underbrace{\frac{\delta_t^{lq}}{1+r_t^{C,J}}}_{\text{market liquidity risk}}. \quad (\text{I.5})$$

The first term on the right-hand side of equation (I.5),  $\frac{r_t^{C,J} - r_t^J}{1+r_t^{C,J}}$ , equals  $(1-\pi)\delta_t^D$  by equation (I.4). This term captures the part of the haircut rate that compensates borrowers' default risk. The second

term,  $\frac{\delta_t^{lq}}{1+r_t^{C,J}}$ , compensates the market liquidity risk.

As explained above, households choose the debt contract at point  $X$  in Figures I.2b and I.2c. Therefore, when state  $U$  is realized in period  $t + J$ , the value of collateral equals the promised payment, and thus, borrowers can pay back the debt either by selling the collateral or by letting lenders sell the collateral. When state  $D$  is realized in period  $t + J$ , borrowers default. In both cases, the termination of the debt contract at maturity in period  $t + J$  does not affect households' budget constraint.<sup>16</sup> This outcome is consistent with the specification of the HASOE model.

This microfoundation sheds light on how to interpret the volatile domestic financial condition presented in Figure 3 through the lens of the HASOE model. Under  $J = 4$ , the spread  $r_t^{C,J} - r_t^J$  corresponds to the gap between the blue solid line and purple dotted line in Figure 3. Through equation (I.5), the volatile fluctuations of the spread can be interpreted as substantial haircut fluctuations due to the borrowers' default risk. Under this interpretation, I incorporate the volatile domestic financial conditions of Peru by augmenting a financial friction shock  $\eta$  to the illiquid asset adjustment cost  $\chi_t(v_{i,t})$ , as discussed in section 4.

## J Details of the Data Construction for the Finance Rate on Consumer Loans

In Figure 3, I plot the time series of the real finance rate on consumer loans in Peru (blue solid line) and compare them with the Peruvian real interest rate in the international financial market (purple dotted line) and the real finance rate on consumer loans in the U.S. (red dashed line). In this appendix section, I provide details of how I construct these data series.

I start by presenting some descriptive statistics regarding the credits in Peru. Figure J.1a presents credit composition by types in Peru during 2002Q4 - 2018Q4 using data from Superintendencia de Banca, Seguros y AFP (SBS). This figure shows that most of the credits in Peru are composed of firm loans (60%–78%). Consumer loans (9%–18%), mortgages (10%–16%) and credit cards (3%–8%) constitute the rest of the credits.

Next, I decompose each type of credit into its denominated currency (Peruvian Sol vs U.S. Dollar) in Figure J.1b, again using data from SBS.<sup>17</sup> Three important observations emerge. First, Peru was a highly dollarized economy: in 2002Q4, 80% of total credits are in dollars. Since 2004, however, the economy has experienced a rapid dedollarization, reaching a 32% dollarized share in 2018Q4. Second, each type of credit has all experienced dedollarization since 2004: between 2004Q1 and 2018Q4, the dollarized share falls from 81% to 42% in firm loans, 38% to 6% in consumer loans, and 96% to 17% in mortgages. Third, even before the dedollarization started in

<sup>16</sup>On the other hand, the initiation of the debt contract in period  $t$  affects their budget constraint by i) the reduction of the illiquid asset position by the amount of collateral  $|v_{i,t}|$  and ii) the associated haircut, which appears as the illiquid asset adjustment cost  $\chi_t(v_{i,t})$ , as discussed above.

<sup>17</sup>Credit card debt is excluded in this figure due to data unavailability.

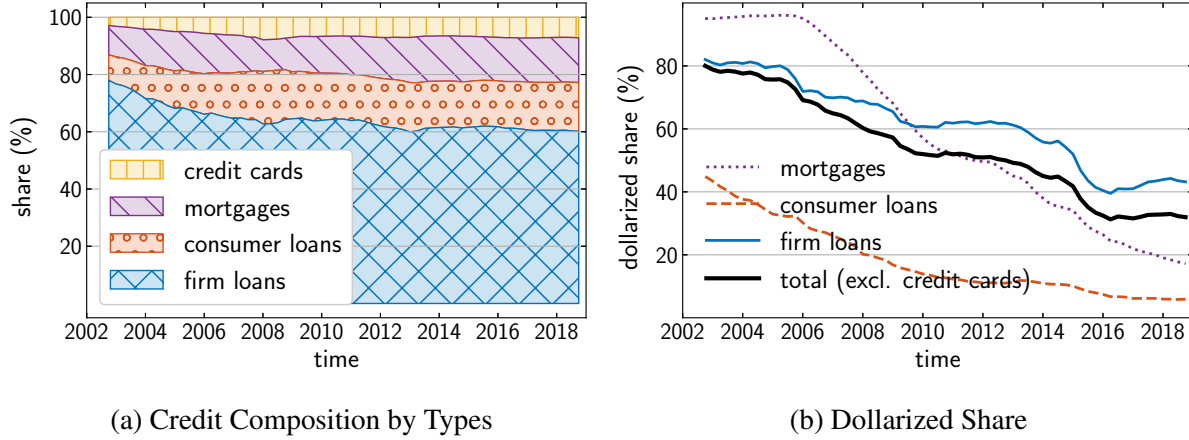


Figure J.1: Credit Composition by Types and Dollarization in Peru

Notes: Data on credit amounts by credit types and denominated currencies in Peru are from SBS.

2004, consumer loans were mostly denominated in Sol.

Motivated by the observations that i) consumer loans were not highly dollarized even before the dedollaration and that ii) the dollarized share in consumer loans has also significantly fallen during the dedollarization starting in 2004, I construct an average finance rate on consumer loans by weight-averaging the rates on dollar consumer loans and on local-currency consumer loans, using each quarter's dollarized share as weights. The weight-averaged finance rates (across denominated currencies) on consumer loans are plotted in Figure 3 as a blue solid line.

The data on the finance rate on consumer loans under each denominated currency (either Peruvian Sol or U.S. Dollar) are obtained from the website of SBS. Two judgment calls are made in choosing and processing the data series. First, SBS provides the finance rate data for consumer loans only with maturity either shorter than or longer than one year (but not with the two maturity categories being merged). Weight-averaging the two series is also not possible because data on their composition is absent. Since household borrowing in the model (or equivalently, cashing out illiquid assets by collateralizing a part of them, as discussed in section 4) is for immediate consumption smoothing rather than running a personal business or making a big investment for a long term, I choose the rate for maturity less than one year. Second, the finance rate data are constructed by averaging finance rates on all consumer loans issued in the last 30 business days, which is approximately 6 calendar weeks. To construct a quarterly series, I process the data as follows: i) I collect the finance rate data on the first available date of each quarter (*i.e.*, the first business day on or after April 1, July 1, October 1, and January 1), whose reference period is the second half of the previous quarter, and the finance rate data on the first available date in the second half of each quarter (*i.e.*, the first business day on or after February 16, May 16, August 16, and November 16), whose the reference period is the first half of the quarter; ii) to boil down two finance rates

(the rates for the first and the second halves of a quarter,  $i_{q1h}$  and  $i_{q2h}$ , respectively) into one, I assume that within each quarter, a half of households borrow during the first half of the quarter and the other half borrow during the latter half of the quarter, and that if  $i_{q1h} > i_{q2h}$ , households who borrowed in the first half of the quarter at  $i_{q1h}$  can refinance their borrowing at  $i_{q2h}$  without any cost; under this assumption, a quarterly finance rate for consumer loans  $i_q$  is constructed by

$$i_q = 0.5 \times \min\{i_{q1h}, i_{q2h}\} + 0.5 \times i_{q2h}.$$

I convert the nominal quarterly finance rates on consumer loans under each denominated currency (either Peruvian Sol or U.S. Dollar) into real rates by deflating them with expected CPI inflation of the denominated currency. The expected inflation is constructed in the same way as in footnote 17.<sup>18</sup> Then, the real finance rates on consumer loans under each denominated currency are weight-averaged across the denominated currencies as explained above.

The real interest rate in the international financial market (purple dotted line in Figure 3), which corresponds to  $r_t$  in the model, is constructed by deflating BCRP's data on lending rates in dollars (TAMEX) with the expected inflation on U.S. CPIs, as described in footnote 17. Note that in the model,  $r_t$  is an interest rate that firms face (through banks) in the international financial market. TAMEX averages interest rates on all types of credits presented in Figure J.1a. The dominant proportion of firm loans in total credits shown in the figure justifies TAMEX as an appropriate data counterpart to  $r_t$  in the model.

The real finance rate on the U.S. consumer loans (the red dashed line in Figure 3) is constructed as follows. First, I obtain the nominal finance rates on personal loans at commercial banks from FRED.<sup>19</sup> Then, I convert the nominal rates to real rates by deflating them with the expected U.S. CPI inflation. In section 4, I compare this series with the real finance rates on Peruvian consumer loans. One caveat in this comparison is that there exists a modest discrepancy between the two series in the maturity of underlying loans, although both series are aimed to capture short-term loans: less than one year for the Peruvian series and exact two years for the U.S. series.

## K Estimation of the HASOE Model under Alternative Assumptions

This section reports the Bayesian estimation results of the HASOE model under each of four alternative assumptions. Table K.1 reports key unconditional business cycle moments and log marginal likelihood, and Table K.2 reports the variance decomposition outcome.

**HASOE** ( $z, \mu, \eta$ ). I estimate the HASOE model without a trend shock  $g$  (*i.e.*, only with  $z$ ,  $\mu$ , and  $\eta$  shocks).

<sup>18</sup>I obtain the Peruvian CPI series from BCRP's website, but it is not clearly stated whether the series is seasonally adjusted. Thus, I seasonally adjust the series using an R package 'seasonal' under a default specification.

<sup>19</sup>The original source of the data is the Board of Governors of the Federal Reserve System, G.19 Consumer Credit.



Table K.1: Unconditional Moments and Marginal Likelihood of the HASOE Model under Alternative Assumptions

	<i>Unconditional Moments</i>				<i>LML</i>
	$\sigma(\Delta \log Y)$	$\sigma(\Delta \log C)$	$\text{corr}[\Delta \log Y, \Delta(TB/Y)]$	$\text{corr}[\Delta \log C, \Delta \log I]$	
HASOE ( $z, \mu, \eta$ )	0.028	0.038	-0.220	-0.184	1147.77
HASOE ( $z, g, \mu, \zeta$ )	0.029	0.039	-0.305	-0.120	1170.03
HASOE ( $z, g, \mu, \nu$ )	0.029	0.037	-0.190	-0.244	1173.80
Counterfactual	0.032	0.039	-0.137	0.017	1127.32
Data	0.027	0.036	-0.346	-0.158	-

*Notes:* Unconditional moments are computed under each posterior draw, and the means across the posterior distribution are reported. ‘LML’ in the sixth column represents log marginal likelihood, which is computed according to Geweke (1999)’s Modified Harmonic Mean method under truncation parameter 0.1. Each model’s log marginal likelihood barely changes over different values of the truncation parameter.

**HASOE ( $z, \mu, \eta, \zeta$ ).** I estimate the HASOE model after replacing the  $\eta$  shock with a preference shock  $\zeta$ , which is imposed on households’ utility as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \frac{C_{i,t}^{1-\gamma}}{1-\gamma} \text{ on workers' utility; } E_0 \sum_{t=0}^{\infty} (\beta_E)^t \zeta_t \frac{(C_t^E)^{1-\gamma}}{1-\gamma} \text{ on entrepreneurs' utility.}$$

**HASOE ( $z, \mu, \eta, \nu$ ).** I estimate the HASOE model after replacing the  $\eta$  shock with an investment shock that Justiniano et al. (2010) study. Specifically, I impose an investment shock  $\nu$  on firms’ investment and capital adjustment cost as follows.

$$\nu_t I_t = K_t - (1 - \delta)K_{t-1}, \quad \nu_t \Phi(K_t, K_{t-1}) = \frac{\phi}{2} \left( \frac{K_t}{K_{t-1}} - g^* \right)^2 K_{t-1}.$$

**Counterfactual Economy** I estimate the counterfactual economy of section 5 using the same data and methods as those used in the estimation of the benchmark economy, HASOE ( $z, g, \mu, \eta$ ).

## L Smoothing

In section 6, I smooth aggregate shocks at the posterior mode to see how my HASOE ( $z, g, \mu, \eta$ ) model depicts the 2008 Peruvian recession. This section explains how the smoothing is actually implemented under the sequence space approach.

For any  $n$ -by- $m$  matrix  $A$ , let  $\text{ravel}(A)$  be an  $(nm)$ -by-1 vector defined as follows:

$$\text{ravel}(A) := [A]_1, [A]_2, \dots, [A]_n]',$$

where  $[A]_i$  is the  $i$ -th row of matrix  $A$ .

Let  $T$  be the truncation length used when solving the model under the sequence space approach,  $T_c(= 0.99T)$  be the truncation length used when evaluating model statistics,<sup>20</sup> and  $T_{obs}(= 155)$

<sup>20</sup>See Online Appendix D for a related discussion.

Table K.2: Variance Decomposition of the HASOE Model under Alternative Assumptions

	$\Delta \log Y_t$	$\Delta \log C_t$	$\Delta \log I_t$	$\Delta(TB_t/Y_t)$
<i>HASOE (z, <math>\mu</math>, <math>\eta</math>) model</i>				
stationary productivity shock (z)	0.990	0.369	0.249	0.059
interest rate shock ( $\mu$ )	0.001	0.023	0.226	0.892
financial friction shock ( $\eta$ )	0.009	0.609	0.525	0.048
<i>HASOE (z, g, <math>\mu</math>, <math>\zeta</math>) model</i>				
stationary productivity shock (z)	0.939	0.374	0.157	0.018
trend shock (g)	0.055	0.068	0.335	0.869
interest rate shock ( $\mu$ )	0.000	0.001	0.005	0.023
preference shock ( $\zeta$ )	0.006	0.558	0.503	0.090
<i>HASOE (z, g, <math>\mu</math>, <math>\nu</math>) model</i>				
stationary productivity shock (z)	0.922	0.317	0.192	0.031
trend shock (g)	0.059	0.072	0.179	0.732
interest rate shock ( $\mu$ )	0.000	0.002	0.007	0.043
investment shock ( $\nu$ )	0.018	0.609	0.622	0.194
<i>Counterfactual model</i>				
stationary productivity shock (z)	0.731	0.136	0.446	0.022
trend shock (g)	0.267	0.712	0.110	0.250
interest rate shock ( $\mu$ )	0.001	0.014	0.337	0.713
financial friction shock ( $\eta$ )	0.001	0.138	0.106	0.015

Notes: The decomposed shares are computed under each posterior draw, and their means are reported.

be the time length of the observed time series  $[\Delta \log Y_t, \Delta \log C_t, \Delta \log I_t, \Delta TB_t/Y_t]$  during 1980Q2-2018Q4. Let  $n_{exo} (= 4)$  be the number of aggregate exogenous variables (*i.e.*,  $z$ ,  $g$ ,  $\mu$ , and  $\eta$ ), and  $n_{obs} (= 4)$  be the number of the observable variables (*i.e.*,  $\Delta \log Y_t$ ,  $\Delta \log C_t$ ,  $\Delta \log I_t$ , and  $\Delta TB_t/Y_t$ ). I define an  $(n_{exo} \times (T_c + T_{obs} - 1))$ -by-1 vector  $\mathbf{E}$  and an  $(n_{obs} \times T_{obs})$ -by-1 vector  $\mathbf{Y}$  as follows.

$$\mathbf{E} := \text{ravel} \left( \begin{bmatrix} \epsilon_{-T_c+1}^z & \epsilon_{-T_c+1}^g & \epsilon_{-T_c+1}^\mu & \epsilon_{-T_c+1}^\eta \\ \vdots & \vdots & \vdots & \vdots \\ \epsilon_0^z & \epsilon_0^g & \epsilon_0^\mu & \epsilon_0^\eta \\ \vdots & \vdots & \vdots & \vdots \\ \epsilon_{T_{obs}-1}^z & \epsilon_{T_{obs}-1}^g & \epsilon_{T_{obs}-1}^\mu & \epsilon_{T_{obs}-1}^\eta \end{bmatrix} \right), \quad \text{and}$$

$$\mathbf{Y} := \text{ravel} \left( \begin{bmatrix} d\Delta \log Y_0 & d\Delta \log C_0 & d\Delta \log I_0 & d\Delta(TB_0/Y_0) \\ \vdots & \vdots & \vdots & \vdots \\ d\Delta \log Y_{T_{obs}-1} & d\Delta \log C_{T_{obs}-1} & d\Delta \log I_{T_{obs}-1} & d\Delta(TB_{T_{obs}-1}/Y_{T_{obs}-1}) \end{bmatrix} \right),$$

where  $d$  is a demeaning operator (*i.e.*, for an observable variable  $OBS_t$  and its long-run average  $\overline{OBS_t}$ ,  $dOBS_t := OBS_t - \overline{OBS_t}$ ). Because each aggregate shock is assumed to follow a normal

distribution, a concatenated vector  $[\mathbf{E}', \mathbf{Y}']'$  follows a multivariate normal distribution. Specifically,

$$\begin{bmatrix} \mathbf{E} \\ \mathbf{Y} \end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{\mathbf{EE}}, & \Sigma_{\mathbf{EY}} \\ \Sigma_{\mathbf{YE}}, & \Sigma_{\mathbf{YY}} \end{bmatrix} \right),$$

where  $\Sigma_{\mathbf{EE}}$  and  $\Sigma_{\mathbf{YY}}$  are the variance-covariance matrices of  $\mathbf{E}$  and  $\mathbf{Y}$ , respectively, and  $\Sigma_{\mathbf{EY}}$  and  $\Sigma_{\mathbf{YE}} (= \Sigma'_{\mathbf{EY}})$  are the covariance matrices between  $\mathbf{Y}$  and  $\mathbf{E}$ .

The relationship between  $\mathbf{E}$  and  $\mathbf{Y}$  can be described as follows.

$$\mathbf{Y} = \Phi \cdot \mathbf{E} + \mathbf{W}, \quad \mathbf{W} \sim N(0, \Sigma_{\mathbf{WW}}), \quad (\text{L.1})$$

where  $\Phi$  is an  $(n_{obs} \times T_{obs})$ -by- $(n_{exo} \times (T_c + T_{obs} - 1))$  matrix whose elements are impulse response coefficients, and  $\mathbf{W}$  is an  $(n_{obs} \times T_{obs})$ -by-1 vector whose elements are measurement errors assumed in the Bayesian estimation.  $\Sigma_{\mathbf{WW}}$  is the variance-covariance matrix of  $\mathbf{W}$ .

Given the parameter values at the posterior mode, we know  $\Sigma_{\mathbf{EE}}$  and  $\Sigma_{\mathbf{WW}}$ . By solving the model at the posterior mode, we obtain the impulse response matrix  $\Phi$ . Then, using equation (L.1), we can compute  $\Sigma_{\mathbf{YY}}$ ,  $\Sigma_{\mathbf{YE}}$ , and  $\Sigma_{\mathbf{EY}}$  as follows.

$$\Sigma_{\mathbf{YY}} = \Phi \cdot \Sigma_{\mathbf{EE}} \cdot \Phi' + \Sigma_{\mathbf{WW}}, \quad \Sigma_{\mathbf{YE}} = \Phi \cdot \Sigma_{\mathbf{EE}}, \quad \text{and} \quad \Sigma_{\mathbf{EY}} = \Sigma'_{\mathbf{YE}}. \quad (\text{L.2})$$

Using equation (L.2), aggregate shocks are smoothed as follows.

$$\mathbf{E}^{sm} := E[\mathbf{E}|\mathbf{Y}] = \Sigma_{\mathbf{EY}} \cdot \Sigma_{\mathbf{YY}}^{-1} \cdot \mathbf{Y} = \Sigma_{\mathbf{EE}} \cdot \Phi' \cdot (\Phi \cdot \Sigma_{\mathbf{EE}} \cdot \Phi' + \Sigma_{\mathbf{WW}})^{-1} \cdot \mathbf{Y}. \quad (\text{L.3})$$

Using the smoothed shocks, I simulate the observable variables in the model as follows.

$$\mathbf{Y}^{sm} := \Phi \mathbf{E}^{sm} = \Phi \Sigma_{\mathbf{EE}} \cdot \Phi' \cdot (\Phi \cdot \Sigma_{\mathbf{EE}} \cdot \Phi' + \Sigma_{\mathbf{WW}})^{-1} \cdot \mathbf{Y}. \quad (\text{L.4})$$

In Figure L.1, I plot the simulated observable variables ( $\mathbf{Y}^{sm}$ ) and their data counterpart ( $\mathbf{Y}$ ), showing that they track each other very closely.  $\mathbf{Y}$  and  $\mathbf{Y}^{sm}$  are not exactly the same only because smoothed measurement errors  $E[\mathbf{W}|\mathbf{Y}]$  are not included in the simulation. Specifically,

$$\mathbf{Y} = E[\mathbf{Y}|\mathbf{Y}] = \Phi \cdot E[\mathbf{E}|\mathbf{Y}] + E[\mathbf{W}|\mathbf{Y}] = \mathbf{Y}^{sm} + E[\mathbf{W}|\mathbf{Y}] \Rightarrow \mathbf{Y} - \mathbf{Y}^{sm} = E[\mathbf{W}|\mathbf{Y}].$$

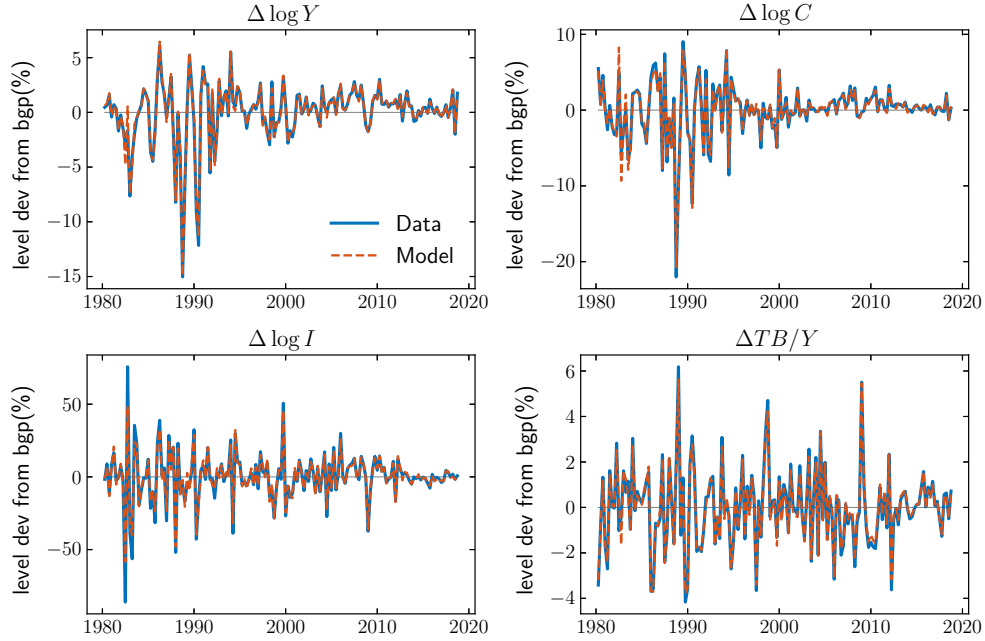


Figure L.1: Simulated Observable Variables using Smoothed Shocks

*Notes:* This figure plots simulated observable variables  $[d\Delta \log Y_t, d\Delta \log C_t, d\Delta \log I_t, d\Delta(TB_t/Y_t)]$  using smoothed shocks (labeled ‘Model’) and their data counterpart (labeled ‘Data’).

## M Additional Figures

**Impulse Responses of Main Drivers to a  $z$  shock.** In Figure M.1, I compare the impulse responses of  $w\bar{l}$  and  $r^a$  to a  $z$  shock between the benchmark and counterfactual economies.

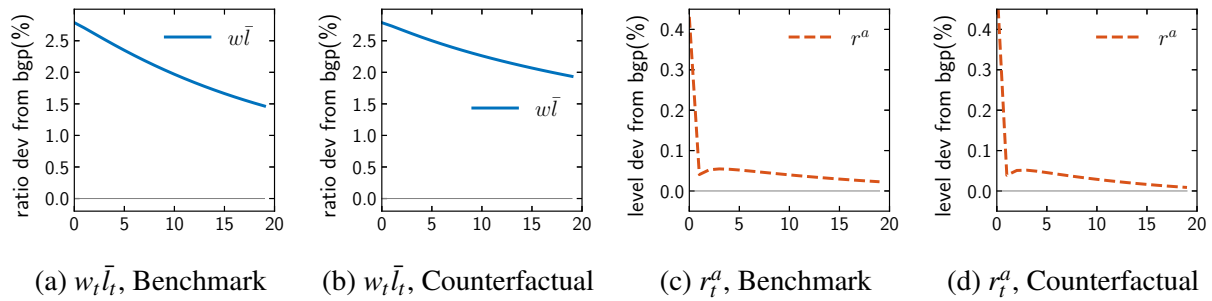
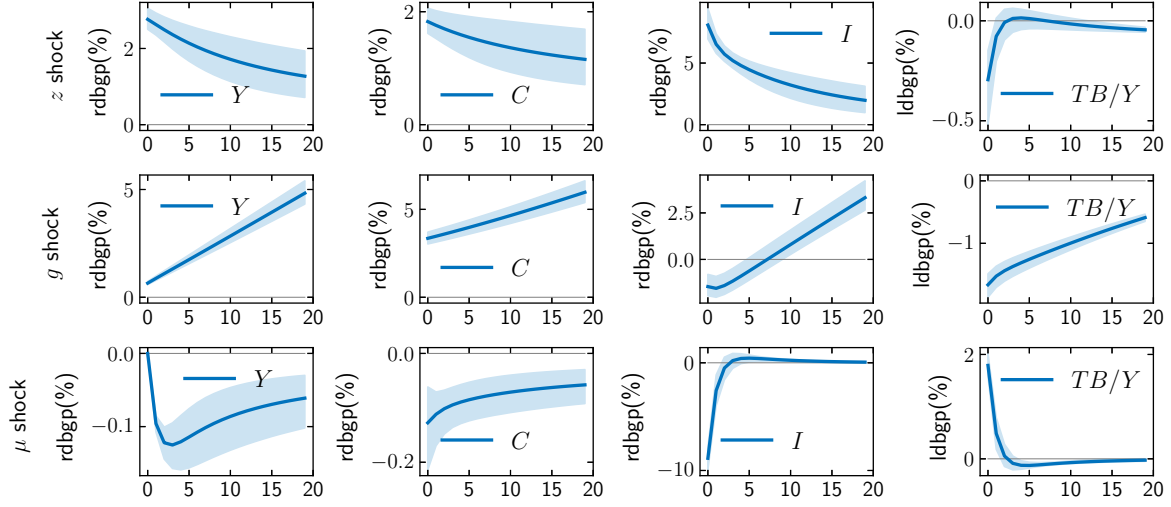


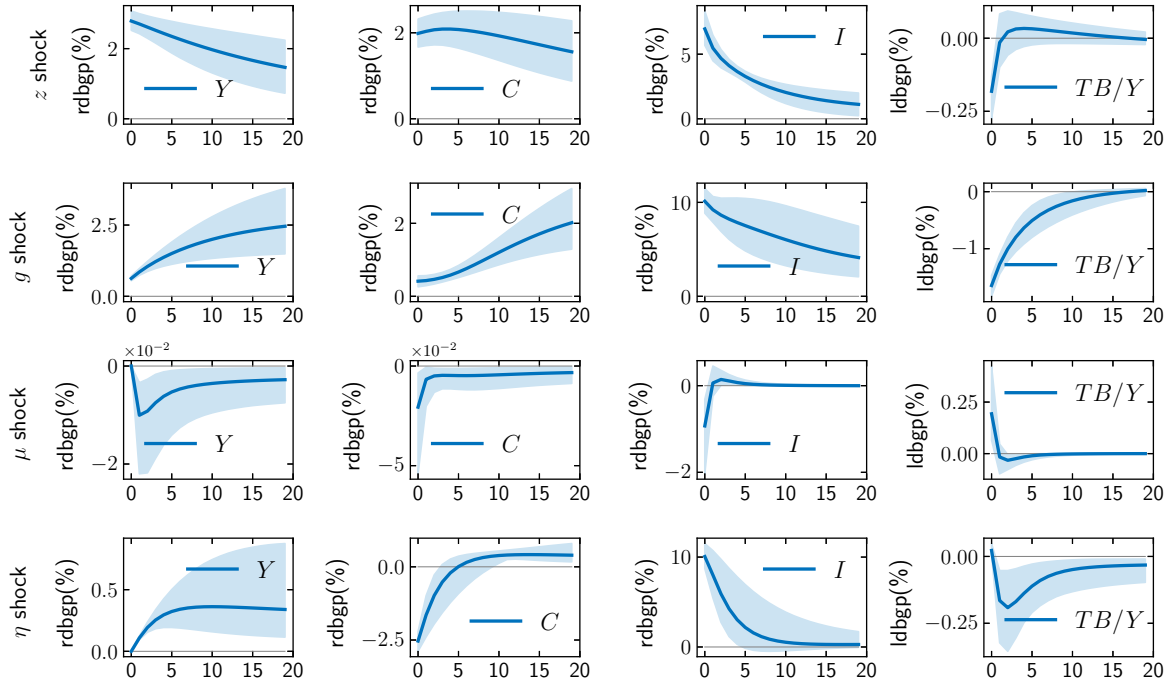
Figure M.1: Impulse Responses of the Main Drivers to a  $z$  Shock: Benchmark vs. Counterfactual

*Notes:* The impulse responses to a one-standard-deviation shock are computed at each posterior draw, and their means across the posterior distribution are plotted.

**Impulse Responses of  $Y$ ,  $C$ ,  $I$ , and  $TB/Y$ .** In Figures M.2a and M.2b, I plot the impulse responses of  $Y$ ,  $C$ ,  $I$ , and  $TB/Y$  to each shock in the RASOE ( $z, g, \mu$ ) and the HASOE ( $z, g, \mu, \eta$ ) models, respectively.



(a) RASOE ( $z, g, \mu$ ) Model



(b) HASOE ( $z, g, \mu, \eta$ ) Model

Figure M.2: Impulse Responses of  $Y$ ,  $C$ ,  $I$ , and  $TB/Y$  to Each Shock

*Notes:* The impulse responses to a one-standard-deviation shock are computed at each posterior draw, and their means across the posterior distribution are plotted. On the y-axis, 'rdbgp(%)' and 'ldbpg(%)' represent a ratio deviation and a level deviation from the balanced growth path (%), respectively. Shaded areas represent 90% credible bands.

**Workers' Consumption ( $C^W$ ) Response in Subgroups.** Figure M.3 plots workers' consumption ( $C^W$ ) response to each aggregate shock within the whole group of workers and within the bottom and the top residual earnings ( $e_{i,t}$ ) deciles in the HASOE ( $z, g, \mu, \eta$ ) model. It also plots the response driven by each driver in workers' budget constraint.

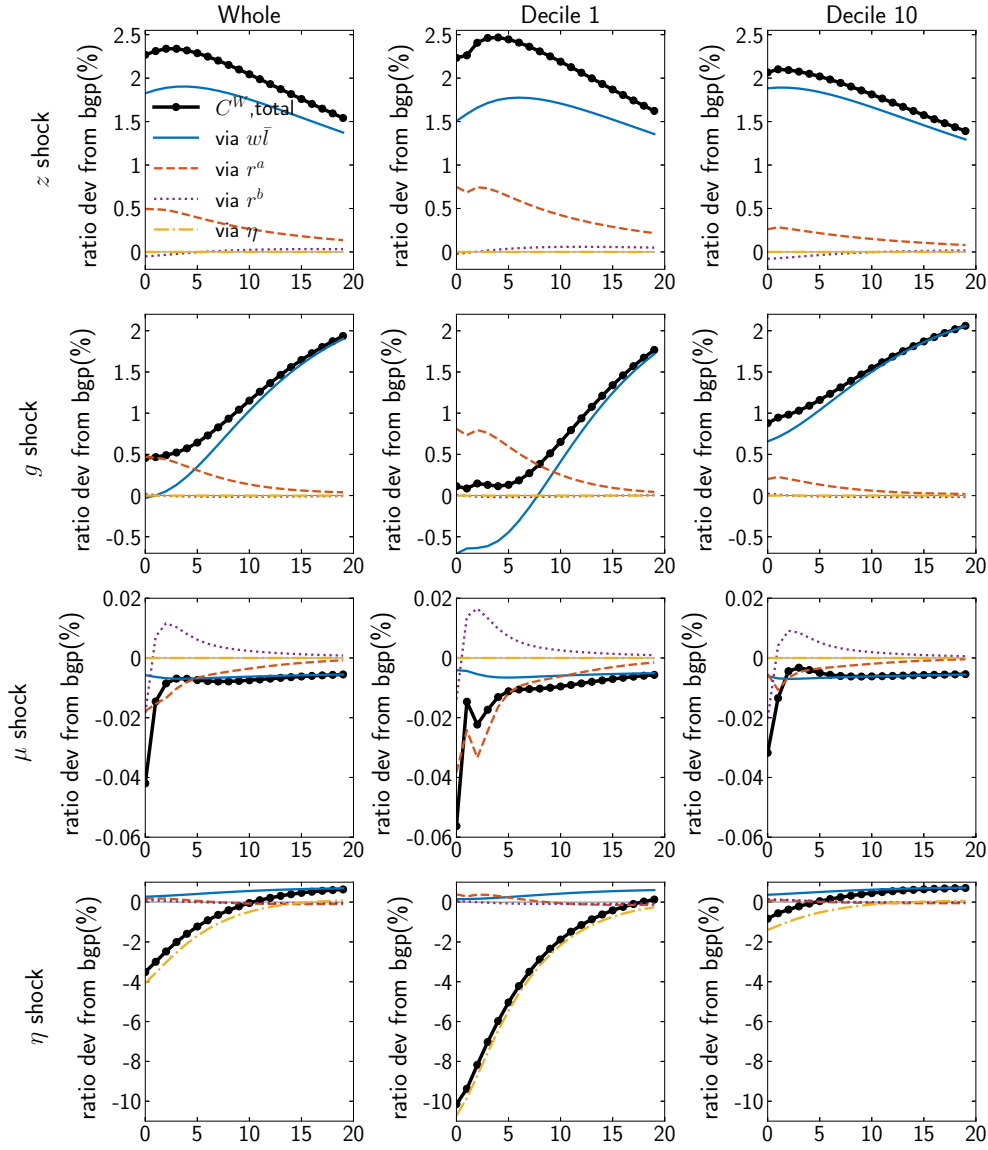


Figure M.3: Workers' Consumption ( $C^W$ ) Response in Subgroups

*Notes:* This figure plots workers' consumption ( $C^W$ ) response to a one-standard-deviation shock within the whole group of workers and within the bottom and the top residual earnings ( $e_{i,t}$ ) deciles in the HASOE ( $z, g, \mu, \eta$ ) model. It also plots the response driven by each driver in workers' budget constraint.

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