[Online Appendix]

Emerging Market Business Cycles with Heterogeneous Agents

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A Details of the RASOE Model

A.1 Equilibrium under Deterministic Paths of Aggregate Exogenous Variables

In this subsection, I characterize the complete set of equilibrium conditions of the RASOE model introduced in subsection 2.1 when the economy is subject to deterministic paths of aggregate exogenous variables $\{z_t, g_t, \mu_t\}_{t=0}^{\infty}$.

Households' optimality conditions are derived as follows.

$$\max_{\{C_t, A_t, L_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta_R)^t \frac{(C_t - \kappa_R X_{t-1} L_t^{1+\omega})^{1-\gamma}}{1-\gamma}$$

$$s.t. \quad C_t + A_t = w_t L_t + (1 + r_t^a) A_{t-1}, \; t \geq 0, \text{ and} \tag{A.1}$$

$$\lim_{j \to \infty} E_t \left[A_{t+j} \Big/ \left(\prod_{s=1}^{j} (1 + r_{t+s}^a) \right) \right] \geq 0.$$

$$\mathfrak{L} = \sum_{t=0}^{\infty} (\beta_R)^t \frac{(C_t - \kappa_R X_{t-1} L_t^{1+\omega})^{1-\gamma}}{1-\gamma} + \sum_{t=0}^{\infty} (\beta_R)^t \lambda_t \left\{ w_t L_t + (1 + r_t^a) A_{t-1} - C_t - A_t \right\}.$$

$$(C_t - \kappa_R X_{t-1} L_t^{1+\omega})^{-\gamma} = \lambda_t, \quad t \geq 0, \tag{A.2}$$

$$\lambda_t = \beta_R (1 + r_{t+1}^a) \lambda_{t+1}, \quad t \geq 0, \quad \text{and} \tag{A.3}$$

$$w_t = (1+\omega) \kappa_R X_{t-1} L_t^{\omega}, \quad t \geq 0. \tag{A.4}$$

Firms' optimality conditions are derived as follows.

$$\max_{\{K_t, F_t, L_t, Y_t, I_t, \Pi_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} Q_{0,t} \Pi_t$$

$$s.t. \quad \Pi_t = Y_t - w_t L_t - I_t - \Phi(K_t, K_{t-1}) + F_t - (1 + r_{t-1}) F_{t-1}, \tag{A.5}$$

$$Y_t = z_t K_{t-1}^{\alpha} (X_t L_t)^{1-\alpha}, \tag{A.6}$$

$$I_t = K_t - (1-\delta) K_{t-1}, \tag{A.7}$$

$$\Phi(K_t, K_{t-1}) = \frac{\phi}{2} \left(\frac{K_t}{K_{t-1}} - g^* \right)^2 K_{t-1},$$

$$Q_{0,t} = \begin{cases} 1 & \text{if} \quad t = 0, \\ 1/(\prod_{s=1}^{t}(1+r_s^a)) & \text{if} \quad t \geq 1, \end{cases} \quad \text{and}$$

$$\lim_{j\to\infty} E_t \left[F_{t+j} \Big/ (\prod_{s=1}^{j}(1+r_{t+s}^a)) \right] \leq 0.$$

$$\Leftrightarrow \max_{\{K_t,F_t,L_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} Q_{0,t} \Bigg[z_t K_{t-1}^{\alpha}(X_t L_t)^{1-\alpha} - w_t L_t - \big(K_t - (1-\delta)K_{t-1}\big) - \frac{\phi}{2}\left(\frac{K_t}{K_{t-1}} - g^*\right)^2 K_{t-1} + F_t - (1+r_{t-1})F_{t-1} \Bigg].$$

$$(1+r_{t+1}^a)\left\{1+\phi\left(\frac{K_t}{K_{t-1}} - g^*\right)\right\} = \alpha z_{t+1}\left(\frac{K_t}{X_{t+1}L_{t+1}}\right)^{\alpha-1} + \left\{1-\delta+\phi\left(\frac{K_{t+1}}{K_t} - g^*\right)\frac{K_{t+1}}{K_t} - \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - g^*\right)^2\right\}, \quad t \geq 0, \tag{A.8}$$

$$1 + r_{t+1}^a = 1 + r_t, \quad t \geq 0, \quad \text{and} \tag{A.9}$$

$$w_t = (1-\alpha) z_t X_t \left(\frac{K_{t-1}}{X_t L_t}\right)^{\alpha}. \tag{A.10}$$

Given the initial conditions on $X_{-1}, A_{-1}, K_{-1}, D_{-1}, F_{-1}$, and r_{-1} and deterministic paths of aggregate exogenous variables $\{z_t, g_t, \mu_t\}_{t=0}^{\infty}$, the equilibrium is characterized by equilibrium variables $\{C_t, A_t, L_t, \lambda_t, K_t, D_t, F_t, w_t, Y_t, I_t, \Pi_t, q_t, r_t^a, r_t\}_{t=0}^{\infty}$ satisfying equations (A.1) - (A.10), (2), (3), (5), and (6).

By Walras' law, we can derive the resource constraint (or, equivalently, the goods market clearing condition in the open economy) using equations (A.1), (2), (3), (A.5), and (5) as follows.

$$\begin{aligned} C_t &= w_t L_t + (1+r_t^a)A_{t-1} - A_t = w_t L_t + (1+r_t^a)q_{t-1} - q_t = w_t L_t + \Pi_t \\ &= Y_t - I_t - \Phi(K_t, K_{t-1}) + F_t - (1+r_{t-1})F_{t-1} = Y_t - I_t - \Phi(K_t, K_{t-1}) + D_t - (1+r_{t-1})D_{t-1}. \end{aligned}$$

$$\therefore C_t + I_t + \frac{\phi}{2}\left(\frac{K_t}{K_{t-1}} - g^*\right)^2 K_{t-1} = Y_t + D_t - (1+r_{t-1})D_{t-1}. \tag{A.11}$$

A.2 Detrended Equilibrium under Deterministic Paths of Aggregate Exogenous Variables

Since the equilibrium characterized in Online Appendix A.1 is nonstationary due to the stochastic trend $\{X_t\}_{t=0}^{\infty}$, we need to detrend the equilibrium to make it stationary. I detrend the equilibrium variables as follows.[1]

$$\tilde{C}_t := C_t / X_{t-1}, \quad \tilde{A}_t := A_t / X_t, \quad \tilde{\lambda}_t := \lambda_t / X_{t-1}^{-\gamma}, \quad \tilde{K}_t := K_t / X_t,$$

[1]Specifically, I detrend flow variables with X_{t-1} and stock variables with X_t.

$$\tilde{D}_t = D_t/X_t, \quad \tilde{F}_t = F_t/X_t, \quad \tilde{w}_t := w_t/X_{t-1}, \quad \tilde{Y}_t := Y_t/X_{t-1},$$

$$\tilde{I}_t := I_t/X_{t-1}, \quad \tilde{\Pi}_t := \Pi_t/X_{t-1}, \quad \tilde{q}_t := q_t/X_t, \quad \text{and} \quad \tilde{TB}_t = TB_t/X_{t-1}.$$

The equilibrium conditions are detrended as follows.

$$\tilde{C}_t + g_t\tilde{A}_t = \tilde{w}_t L_t + (1+r_t^a)\tilde{A}_{t-1}, \quad t \geq 0, \tag{A.12}$$

$$(\tilde{C}_t - \kappa_R L_t^{1+\omega})^{-\gamma} = \tilde{\lambda}_t, \quad t \geq 0, \tag{A.13}$$

$$\tilde{\lambda}_t = g_t^{-\gamma}\beta_R(1+r_t)\tilde{\lambda}_{t+1}, \quad t \geq 0, \tag{A.14}$$

$$\tilde{w}_t = (1+\omega)\kappa_R L_t^{\omega}, \quad t \geq 0, \tag{A.15}$$

$$\tilde{\Pi}_t = \tilde{Y}_t - \tilde{w}_t L_t - \tilde{I}_t - \frac{\phi}{2}\left(\frac{\tilde{K}_t}{\tilde{K}_{t-1}}g_t - g^*\right)^2\tilde{K}_{t-1} + g_t\tilde{F}_t - (1+r_{t-1})\tilde{F}_{t-1}, \quad t \geq 0, \tag{A.16}$$

$$\tilde{Y}_t = z_t g_t^{1-\alpha}\tilde{K}_{t-1}^{\alpha}L_t^{1-\alpha}, \quad t \geq 0, \tag{A.17}$$

$$\tilde{I}_t = g_t\tilde{K}_t - (1-\delta)\tilde{K}_{t-1}, \quad t \geq 0, \tag{A.18}$$

$$\begin{aligned}(1+r_t)\left\{1+\phi\left(\frac{\tilde{K}_t}{\tilde{K}_{t-1}}g_t - g^*\right)\right\} &= \alpha z_{t+1}g_{t+1}^{1-\alpha}\left(\frac{\tilde{K}_t}{L_{t+1}}\right)^{\alpha-1} \\ &+ \left\{1-\delta+\phi\left(\frac{\tilde{K}_{t+1}}{\tilde{K}_t}g_{t+1} - g^*\right)\frac{\tilde{K}_{t+1}}{\tilde{K}_t}g_{t+1} - \frac{\phi}{2}\left(\frac{\tilde{K}_{t+1}}{\tilde{K}_t}g_{t+1} - g^*\right)^2\right\}, \quad t \geq 0,\end{aligned} \tag{A.19}$$

$$1 + r_{t+1}^a = 1 + r_t, \quad t \geq 0, \tag{A.20}$$

$$\tilde{w}_t = (1-\alpha)z_t g_t^{1-\alpha}\left(\frac{\tilde{K}_{t-1}}{L_t}\right)^{\alpha}, \quad t \geq 0, \tag{A.21}$$

$$\tilde{A}_t = \tilde{q}_t, \quad t \geq 0, \tag{A.22}$$

$$1 + r_t^a = \frac{\tilde{\Pi}_t + g_t\tilde{q}_t}{\tilde{q}_{t-1}}, \quad t \geq 0, \tag{A.23}$$

$$\tilde{F}_t = \tilde{D}_t, \quad t \geq 0, \quad \text{and} \tag{A.24}$$

$$r_t = r^* + \psi\left\{\exp\left(\frac{\tilde{D}_t - \tilde{D}^*}{\tilde{Y}^*}\right) - 1\right\} - \theta_z(z_t - 1) - \theta_g\left(\frac{g_t}{g^*} - 1\right) + \mu_t - 1, \quad t \geq 0. \tag{A.25}$$

Given the initial conditions on $\tilde{A}_{-1}, \tilde{K}_{-1}, \tilde{D}_{-1}, \tilde{F}_{-1}$, and r_{-1} and deterministic paths of aggregate exogenous variables $\{z_t, g_t, \mu_t\}_{t=0}^{\infty}$, the detrended equilibrium is characterized by equilibrium variables $\{\tilde{C}_t, \tilde{A}_t, L_t, \tilde{\lambda}_t, \tilde{K}_t, \tilde{D}_t, \tilde{F}_t, \tilde{w}_t, \tilde{Y}_t, \tilde{I}_t, \tilde{\Pi}_t, \tilde{q}_t, r_t^a, r_t\}_{t=0}^{\infty}$ satisfying equations (A.12) - (A.25).

The resource constraint and the equation for the trade balance are also detrended as follows.

$$\tilde{C}_t + \tilde{I}_t + \frac{\phi}{2}\left(\frac{\tilde{K}_t}{\tilde{K}_{t-1}}g_t - g^*\right)^2\tilde{K}_{t-1} = \tilde{Y}_t + g_t\tilde{D}_t - (1+r_{t-1})\tilde{D}_{t-1}, \quad t \geq 0. \tag{A.26}$$

$$\tilde{TB}_t = -g_t\tilde{D}_t + (1+r_{t-1})\tilde{D}_{t-1}, \quad t \geq 0. \tag{A.27}$$

A.3 Steady State of the Detrended Equilibrium

For any variable x_t, let x_{ss} denote its steady state value. In the steady state of the detrended equilibrium, the following equations hold.

$$z_{ss} = 1, \quad g_{ss} = g^*, \quad \text{and} \quad \mu_{ss} = 1.$$
$$\tilde{D}_{ss} = \tilde{D}^* \quad \text{and} \quad \tilde{Y}_{ss} = \tilde{Y}^* \quad \text{(by definition).}$$

In the steady state, the detrended equilibrium conditions become

$$\tilde{C}_{ss} = \tilde{w}_{ss}L_{ss} + (1 + r^a_{ss} - g_{ss})\tilde{A}_{ss}, \tag{A.28}$$

$$(\tilde{C}_{ss} - \kappa_R L_{ss}^{1+\omega})^{-\gamma} = \tilde{\lambda}_{ss}, \tag{A.29}$$

$$g_{ss}^{-\gamma}\beta_R(1+r_{ss}) = 1, \tag{A.30}$$

$$\tilde{w}_{ss} = (1+\omega)\kappa_R L_{ss}^{\omega}, \tag{A.31}$$

$$\tilde{\Pi}_{ss} = \tilde{Y}_{ss} - \tilde{w}_{ss}L_{ss} - \tilde{I}_{ss} - (1 + r_{ss} - g_{ss})\tilde{F}_{ss}, \tag{A.32}$$

$$\tilde{Y}_{ss} = z_{ss}g_{ss}^{1-\alpha}\tilde{K}_{ss}^{\alpha}L_{ss}^{1-\alpha}, \tag{A.33}$$

$$\tilde{I}_{ss} = (g_{ss} - 1 + \delta)\tilde{K}_{ss}, \tag{A.34}$$

$$r_{ss} + \delta = \alpha(\tilde{Y}_{ss}/\tilde{K}_{ss}), \tag{A.35}$$

$$r^a_{ss} = r_{ss}, \tag{A.36}$$

$$\tilde{w}_{ss} = (1-\alpha)z_{ss}g_{ss}^{1-\alpha}(\tilde{K}_{ss}/L_{ss})^{\alpha}, \tag{A.37}$$

$$\tilde{A}_{ss} = \tilde{q}_{ss}, \tag{A.38}$$

$$(1 + r^a_{ss} - g_{ss})\tilde{q}_{ss} = \tilde{\Pi}_{ss}, \tag{A.39}$$

$$\tilde{F}_{ss} = \tilde{D}_{ss}, \quad \text{and} \tag{A.40}$$

$$r_{ss} = r^*. \tag{A.41}$$

Moreover, the resource constraint and the equation for the trade balance become

$$\tilde{C}_{ss} + \tilde{I}_{ss} = \tilde{Y}_{ss} - (1 + r_{ss} - g_{ss})\tilde{D}_{ss}, \quad \text{and} \tag{A.42}$$

$$\tilde{TB}_{ss} = (1 + r_{ss} - g_{ss})\tilde{D}_{ss}. \tag{A.43}$$

Using equations (A.38), (A.39), (A.32), (A.37), (A.33), (A.35),(A.34), (A.40), and (A.36), we can derive a relationship among stock variables $\tilde{A}_{ss}$, $\tilde{K}_{ss}$, and $\tilde{D}_{ss}$ as follows.

$$\begin{aligned}(1+r^a_{ss}-g_{ss})\tilde{A}_{ss} &= (1+r^a_{ss}-g_{ss})\tilde{q}_{ss} = \tilde{\Pi}_{ss} = \tilde{Y}_{ss} - \tilde{w}_{ss}L_{ss} - \tilde{I}_{ss} - (1+r_{ss}-g_{ss})\tilde{F}_{ss} \\ &= \alpha\tilde{Y}_{ss} - \tilde{I}_{ss} - (1+r_{ss}-g_{ss})\tilde{F}_{ss} = (r_{ss}+\delta)\tilde{K}_{ss} - \tilde{I}_{ss} - (1+r_{ss}-g_{ss})\tilde{F}_{ss} \\ &= (r_{ss}+\delta)\tilde{K}_{ss} - (g_{ss}-1+\delta)\tilde{K}_{ss} - (1+r_{ss}-g_{ss})\tilde{F}_{ss} \\ &= (r_{ss}+\delta)\tilde{K}_{ss} - (g_{ss}-1+\delta)\tilde{K}_{ss} - (1+r_{ss}-g_{ss})\tilde{D}_{ss} \\ &= (1+r_{ss}-g_{ss})(\tilde{K}_{ss}-\tilde{D}_{ss}) = (1+r^a_{ss}-g_{ss})(\tilde{K}_{ss}-\tilde{D}_{ss}).\end{aligned}$$

$$\therefore \tilde{A}_{ss} = \tilde{K}_{ss} - \tilde{D}_{ss}.$$

A.4 An Equivalent, Centralized Economy

In subsection 2.1, I present a decentralized version of a stanrard RASOE model. In this subsection, I present an equivalent, centralized version of the RASOE model, which appears far more frequently in related studies.

Consider an economy composed of representative households. They produce output Y_t using capital K_{t-1} and their own labor L_t, make investment I_t to accumulate capital, and borrow debt D_t from the international financial market. They optimize the GHH preference subject to resource constraints and the no-Ponzi-game constraint as follows.

$$\max_{\{C_t,I_t,K_t,Y_t,D_t,L_t\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} (\beta_R)^t \frac{(C_t - \kappa_R X_{t-1} L_t^{1+\omega})^{1-\gamma}}{1-\gamma}$$

$$s.t. \quad C_t + I_t + \frac{\phi}{2}\left(\frac{K_t}{K_{t-1}} - g^*\right)^2 K_{t-1} = Y_t + D_t - (1+r_{t-1})D_{t-1}, \tag{A.44}$$

$$Y_t = z_t K_{t-1}^{\alpha} (X_t L_t)^{1-\alpha}, \tag{A.45}$$

$$I_t = K_t - (1-\delta)K_{t-1}, \text{ and} \tag{A.46}$$

$$\lim_{j\to\infty} E_t \left[D_{t+j} \Big/ \left(\prod_{s=0}^{j-1}(1+r_{t+s})\right)\right] \leq 0.$$

The interest rate r_t in the international financial market is determined according to equation (6). As in Online Appendix A.1, I derive the complete set of equilibrium conditions when the economy is subject to deterministic paths of aggregate exogenous variables $\{z_t, g_t, \mu_t\}_{t=0}^{\infty}$.

$$\mathfrak{L} = \sum_{t=0}^{\infty} (\beta_R)^t \frac{(C_t - \kappa_R X_{t-1} L_t^{1+\omega})^{1-\gamma}}{1-\gamma}$$
$$+ \sum_{t=0}^{\infty} (\beta_R)^t \lambda_t \left\{ z_t K_{t-1}^{\alpha} (X_t L_t)^{1-\alpha} + D_t - (1+r_{t-1}) D_{t-1} - C_t - (K_t - (1-\delta) K_{t-1}) - \frac{\phi}{2} \left(\frac{K_t}{K_{t-1}} - g^* \right)^2 K_{t-1} \right\}.$$

$$(C_t - \kappa_R X_{t-1} L_t^{1+\omega})^{-\gamma} = \lambda_t, \quad t \geq 0, \tag{A.47}$$

$$\lambda_t = \beta_R (1+r_t) \lambda_{t+1}, \quad t \geq 0, \tag{A.48}$$

$$(1-\alpha) z_t X_t \left(\frac{K_{t-1}}{X_t L_t} \right)^{\alpha} = (1+\omega) \kappa_R X_{t-1} L_t^{\omega}, \quad t \geq 0, \quad \text{and} \tag{A.49}$$

$$\begin{aligned} (1+r_t) \left\{ 1 + \phi \left(\frac{K_t}{K_{t-1}} - g^* \right) \right\} &= \alpha z_{t+1} \left(\frac{K_t}{X_{t+1} L_{t+1}} \right)^{\alpha-1} \\ &+ \left\{ 1 - \delta + \phi \left(\frac{K_{t+1}}{K_t} - g^* \right) \frac{K_{t+1}}{K_t} - \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - g^* \right)^2 \right\}, \quad t \geq 0. \end{aligned} \tag{A.50}$$

Given the initial conditions on X_{-1}, K_{-1}, D_{-1}, and r_{-1} and deterministic paths of aggregate exogenous variables $\{z_t, g_t, \mu_t\}_{t=0}^{\infty}$, the equilibrium is characterized by equilibrium variables $\{C_t, L_t, \lambda_t, K_t, D_t, Y_t, I_t, r_t\}_{t=0}^{\infty}$ satisfying equations (A.44) - (A.50) and (6).

It is straightforward to verify that i) all the equilibrium conditions in the centralized economy are satisfied in the decentralized economy and that ii) $\{A_t, F_t, w_t, \Pi_t, q_t, r_t^a\}_{t=0}^{\infty}$ can be constructed in the centralized economy such that the equilibrium variables and the newly constructed variables of the centralized economy together satisfy all the equilibrium conditions of the decentralized economy. Therefore, the decentralized economy in subsection 2.1 and the centralized economy in this subsection yield the same equilibrium.

B Details of the HASOE Model

B.1 Equilibrium under Deterministic Paths of Aggregate Exogenous Variables

In this subsection, I characterize the complete set of equilibrium conditions of the HASOE model in subsection 4 when the economy is subject to deterministic paths of aggregate exogenous variables $\{z_t, g_t, \mu_t, \eta_t\}_{t=0}^{\infty}$. The HASOE model in subsection 2.2, which does not have a financial friction shock η_t, has the same equilibrium conditions except that η_t should be replaced with 1.

Workers' problem can be expressed as the following Bellman equation.

$$V_t^W(e_1,e_2,b_-,a_-;\Gamma) = \max_{c,b,a} \frac{c^{1-\gamma}}{1-\gamma} + \beta \sum_{e_1',e_2'} P(e_1',e_2'|e_1,e_2) V_{t+1}^W(e_1',e_2',b,a;\Gamma)$$

$$s.t. \quad c+b+a+\eta_t\chi_t(a-(1+r_t^a)a_-,a_-;\Gamma) = w_t\Gamma e\bar{l}_t + (1-\xi)(1+r_t^b)b_- + (1+r_t^a)a_-,$$

$$a \geq 0, \quad b \geq 0, \quad \text{and}$$

$$\log e = \log e_1 + \log e_2.$$

On the balanced growth path where $z_t = 1$, $g_t = g^*$, $\mu_t = 1$,and $\eta_t = 1$, V_t^W grows at the rate of $(g^*)^{1-\gamma}$ or, equivalently, $V_{t+1}^W = (g^*)^{1-\gamma}\, V_t^W$.

Under the parametrization of $\chi_t(v,a_-;\Gamma)$ in subsection 2.2, its first-order derivatives are

$$\chi_{1,t}(v,a_-;\Gamma) = sign(v)\ \chi_1\chi_2 \left| \frac{v}{(1+r_t^a)a_- + \chi_0\Upsilon(\Gamma)X_{t-1}} \right|^{\chi_2-1}, \quad \text{and}$$

$$\chi_{2,t}(v,a_-;\Gamma) = \chi_1(1-\chi_2) \left| \frac{v}{(1+r_t^a)a_- + \chi_0\Upsilon(\Gamma)X_{t-1}} \right|^{\chi_2} (1+r_t^a).$$

Both $\chi_{1,t}(v,a_-;\Gamma)$ and $\chi_{2,t}(v,a_-;\Gamma)$ are continuous in (v,a_-) everywhere, including the area around $v=0$. Therefore, $\chi_t(v,a_-;\Gamma)$ is continuous and differentiable in (v,a_-) everywhere.

Workers' optimality conditions are derived as follows.

$$\begin{aligned} V_t^W(e_1,e_2,b_-,a_-;\Gamma) = \max_{c,b,a}\ & \frac{c^{1-\gamma}}{1-\gamma} + \beta \sum_{e_1',e_2'} P(e_1',e_2'|e_1,e_2) V_{t+1}^W(e_1',e_2',b,a;\Gamma) \\ & + \lambda\{w_t\Gamma e\bar{l}_t + (1-\xi)(1+r_t^b)b_- + (1+r_t^a)a_- \\ & - c - b - a - \eta_t\chi_t(a-(1+r_t^a)a_-,a_-;\Gamma)\} + \varphi^b b + \varphi^a a. \end{aligned}$$

$$\lambda = c^{-\gamma}, \tag{B.1}$$

$$\lambda = \beta \sum_{e_1',e_2'} P(e_1',e_2'|e_1,e_2) V_{b,t+1}^W(e_1',e_2',b,a;\Gamma) + \varphi^b, \tag{B.2}$$

$$\lambda\{1+\eta_t\chi_{1,t}(a-(1+r_t^a)a_-,a_-;\Gamma)\} = \beta \sum_{e_1',e_2'} P(e_1',e_2'|e_1,e_2) V_{a,t+1}^W(e_1',e_2',b,a;\Gamma) + \varphi^a, \tag{B.3}$$

$$V_{b,t}^W(e_1,e_2,b_-,a_-;\Gamma) = (1-\xi)(1+r_t^b)\lambda, \tag{B.4}$$

$$\begin{aligned} V_{a,t}^W(e_1,e_2,b_-,a_-;\Gamma) = \lambda\{&(1+r_t^a) + (1+r_t^a)\eta_t\chi_{1,t}(a-(1+r_t^a)a_-,a_-;\Gamma) \\ &- \eta_t\chi_{2,t}(a-(1+r_t^a)a_-,a_-;\Gamma)\}, \end{aligned} \tag{B.5}$$

$$c+b+a+\eta_t\chi_t(a-(1+r_t^a)a_-,a_-;\Gamma) = w_t\Gamma e\bar{l}_t + (1-\xi)(1+r_t^b)b_- + (1+r_t^a)a_-, \tag{B.6}$$

$$\varphi^b \geq 0, \quad b \geq 0, \quad \varphi^b b = 0, \quad \text{and} \tag{B.7}$$

$$\varphi^a \geq 0, \quad a \geq 0, \quad \varphi^a a = 0. \tag{B.8}$$

Entrepreneurs' optimality conditions are derived as follows.

$$\begin{aligned} &\max_{\{C_t^E, A_t^E\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta_E)^t \frac{(C_t^E)^{1-\gamma}}{1-\gamma} \\ s.t. \quad & C_t^E + A_t^E = R_t^E + (1+r_t^a)A_{t-1}^E, \quad \text{and} \\ & \lim_{j\to\infty} E_t \left[A_{t+j}^E \Big/ (\prod_{s=1}^{j}(1+r_{t+s}^a)) \right] \geq 0. \end{aligned} \tag{B.9}$$

$$\mathfrak{L} = \sum_{t=0}^{\infty} (\beta_E)^t \frac{(C_t^E)^{1-\gamma}}{1-\gamma} + \sum_{t=0}^{\infty} (\beta_E)^t \lambda_t^E \{R_t^E + (1+r_t^a)A_{t-1}^E - C_t^E - A_t^E\}.$$

$$\lambda_t^E = (C_t^E)^{-\gamma}, \quad t \geq 0,$$

$$\lambda_t^E = \beta_E (1+r_{t+1}^a)\lambda_{t+1}^E, \quad t \geq 0$$

$$\Rightarrow (C_t^E)^{-\gamma} = \beta_E (1+r_{t+1}^a)(C_{t+1}^E)^{-\gamma}, \quad t \geq 0. \tag{B.10}$$

Firms' optimality conditions are the same as those derived in Online Appendix A.1.

Given the initial conditions on $\Psi_0(e_1, e_2, b_-, a_-|\Gamma)$, X_{-1}, A_{-1}, A_{-1}^E, K_{-1}, D_{-1}, B_{-1}, F_{-1}, and r_{-1} and deterministic paths of aggregate exogenous variables $\{z_t, g_t, \mu_t, \eta_t\}_{t=0}^{\infty}$, *(i)* individual workers' policy functions $\{c_t(e_1,e_2,b_-,a_-;\Gamma), b_t(e_1,e_2,b_-,a_-;\Gamma), a_t(e_1,e_2,b_-,a_-;\Gamma)\}_{t=0}^{\infty}$, first-order derivatives of the value functions $\{V_{b,t}^W(e_1,e_2,b_-,a_-;\Gamma), V_{a,t}^W(e_1,e_2,b_-,a_-;\Gamma)\}_{t=0}^{\infty}$, and Lagrangian multipliers $\{\lambda_t(e_1,e_2,b_-,a_-;\Gamma), \varphi_t^b(e_1,e_2,b_-,a_-;\Gamma), \varphi_t^a(e_1,e_2,b_-,a_-;\Gamma)\}_{t=0}^{\infty}$ that satisfy workers' optimality conditions (B.1) - (B.8), *(ii)* conditional cumulative distributions $\{\Psi_t(e_1,e_2,b_-,a_-|\Gamma)\}_{t=1}^{\infty}$ that evolve over time according to equation (13), and *(iii)* prices and aggregate variables $\{r_t^b, r_t^a, r_t, w_t, q_t, \bar{l}_t, L_t, \Pi_t, Y_t, I_t, K_t, F_t, D_t, TB_t, C_t, C_t^E, A_t, A_t^E, R_t^E, B_t, \chi_t^{agg}\}_{t=0}^{\infty}$ satisfying entrepreneurs' optimality conditions (B.9) and (B.10), firms' optimality conditions (A.5) - (A.10), aggregation equations (18), (19), (20), and (23), and other equilibrium conditions (2), (3), (6), (8), (11), (12), (15), (16), and (17) constitute the equilibrium of the economy.

By Walras' law, we can show that the resource constraint (A.11) (or, equivalently, the goods market clearing condition in the open economy) also holds in the HASOE model as follows. By aggregating workers' budget constraint (B.6), we obtain

$$C_t^W + B_t^W + A_t^W + \chi_t^W = w_t(L_t/p) + (1-\xi)(1+r_t^b)B_{t-1}^W + (1+r_t^a)A_{t-1}^W. \tag{B.11}$$

Combining equation (B.11) with equations (B.9), (17), (18), (19), (20), and (23), we can obtain

$$C_t + B_t + A_t = w_t L_t + (1 + r_t^b) B_{t-1} + (1 + r_t^a) A_{t-1}. \tag{B.12}$$

Combining equations (B.12), (2), (3), (A.5), (16), and (15), we can obtain

$$\begin{aligned} C_t &= w_t L_t + (1+r_t^b)B_{t-1} - B_t + (1+r_t^a)A_{t-1} - A_t = w_t L_t + (1+r_t^b)B_{t-1} - B_t + (1+r_t^a)q_{t-1} - q_t \\ &= w_t L_t + (1+r_t^b)B_{t-1} - B_t + \Pi_t = Y_t - I_t - \Phi(K_t, K_{t-1}) + F_t - (1+r_{t-1})F_{t-1} + (1+r_t^b)B_{t-1} - B_t \\ &= Y_t - I_t - \Phi(K_t, K_{t-1}) + D_t - (1+r_{t-1})D_{t-1}. \end{aligned}$$

$$\therefore C_t + I_t + \frac{\phi}{2}\left(\frac{K_t}{K_{t-1}} - g^*\right)^2 K_{t-1} = Y_t + D_t - (1+r_{t-1})D_{t-1}.$$

B.2 Detrended Equilibrium under Deterministic Paths of Aggregate Exogenous Variables

Since the equilibrium characterized in Online Appendix B.1 is nonstationary due to the stochastic trend $\{X_t\}_{t=0}^{\infty}$, we need to detrend the equilibrium to make it stationary. I detrend the variables and functions as follows.[2]

$$\tilde{c}_{i,t} := c_{i,t}/X_{t-1}, \quad \tilde{b}_{i,t} := b_{i,t}/X_t, \quad \tilde{a}_{i,t} := a_{i,t}/X_t,$$

$$\tilde{\lambda}_{i,t} := \lambda_{i,t}/X_{t-1}^{-\gamma}, \quad \tilde{\varphi}_{i,t}^b := \varphi_{i,t}^b/X_{t-1}^{-\gamma}, \quad \tilde{\varphi}_{i,t}^a := \varphi_{i,t}^a/X_{t-1}^{-\gamma},$$

$$\tilde{Y}_t := Y_t/X_{t-1}, \quad \tilde{C}_t := C_t/X_{t-1}, \quad \tilde{C}_t^E := C_t^E/X_{t-1}, \quad \tilde{C}_t^W := C_t^W/X_{t-1}, \quad \tilde{I}_t := I_t/X_{t-1}, \quad \tilde{R}_t^E := R_t^E/X_{t-1},$$

$$\tilde{\chi}_t^{agg} := \chi_t^{agg}/X_{t-1}, \quad \tilde{\chi}_t^W := \chi_t^W/X_{t-1}, \quad \tilde{w}_t := w_t/X_{t-1}, \quad \tilde{\Pi}_t := \Pi_t/X_{t-1}, \quad \tilde{TB}_t := TB_t/X_{t-1},$$

$$\tilde{B}_t := B_t/X_t, \quad \tilde{B}_t^W := B_t^W/X_t, \quad \tilde{A}_t := A_t/X_t, \quad \tilde{A}_t^E := A_t^E/X_t, \quad \tilde{A}_t^W := A_t^W/X_t,$$

$$\tilde{q}_t := q_t/X_t, \quad \tilde{D}_t := D_t/X_t, \quad \tilde{K}_t := K_t/X_t, \quad \text{and} \quad \tilde{F}_t := F_t/X_t.$$

$$\tilde{\Psi}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_- | \Gamma) := \Psi_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1} | \Gamma),$$

$$\tilde{V}_{b,t}^W(e_1, e_2, \tilde{b}_-, \tilde{a}_-; \Gamma) := V_{b,t}^W(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1}; \Gamma)/X_{t-1}^{-\gamma},$$

$$\tilde{V}_{a,t}^W(e_1, e_2, \tilde{b}_-, \tilde{a}_-; \Gamma) := V_{a,t}^W(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1}; \Gamma)/X_{t-1}^{-\gamma},$$

$$\tilde{c}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-; \Gamma) := c_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1}; \Gamma)/X_{t-1},$$

$$\tilde{b}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-; \Gamma) := b_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1}; \Gamma)/X_t,$$

$$\tilde{a}_t(e_1, e_2, \tilde{b}_-, \tilde{a}_-; \Gamma) := a_t(e_1, e_2, \tilde{b}_- X_{t-1}, \tilde{a}_- X_{t-1}; \Gamma)/X_t,$$

[2] As in Online Appendix A.2, I detrend flow variables with X_{t-1} and stock variables with X_t.

$$\tilde{\lambda}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma) := \lambda_t(e_1,e_2,\tilde{b}_- X_{t-1},\tilde{a}_- X_{t-1};\Gamma)/X_{t-1}^{-\gamma},$$

$$\tilde{\varphi}_t^b(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma) := \varphi_t^b(e_1,e_2,\tilde{b}_- X_{t-1},\tilde{a}_- X_{t-1};\Gamma)/X_{t-1}^{-\gamma},$$

$$\tilde{\varphi}_t^a(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma) := \varphi_t^a(e_1,e_2,\tilde{b}_- X_{t-1},\tilde{a}_- X_{t-1};\Gamma)/X_{t-1}^{-\gamma}, \quad \text{and}$$

$$\tilde{\chi}_t(\tilde{v},\tilde{a}_-;\Gamma) := \chi_1\left|\frac{\tilde{v}}{(1+r_t^a)\tilde{a}_- + \chi_0\Upsilon(\Gamma)}\right|^{\chi_2}\big((1+r_t^a)\tilde{a}_- + \chi_0\Upsilon(\Gamma)\big), \quad \text{where} \quad \Upsilon(\Gamma) = \tilde{w}_{ss}\Gamma\bar{e}\bar{l}_{ss}.$$

($\tilde{w}_{ss}$ and $\bar{l}_{ss}$ are the steady state values of $\tilde{w}_t$ and $\bar{l}_t$, respectively.)

The first-order derivatives of $\tilde{\chi}_t(\tilde{v},\tilde{a}_-;\Gamma)$ are

$$\tilde{\chi}_{1,t}(\tilde{v},\tilde{a}_-;\Gamma) = sign(\tilde{v})\chi_1\chi_2\left|\frac{\tilde{v}}{(1+r_t^a)\tilde{a}_- + \chi_0\Upsilon(\Gamma)}\right|^{\chi_2-1}, \quad \text{and}$$

$$\tilde{\chi}_{2,t}(\tilde{v},\tilde{a}_-;\Gamma) = \chi_1(1-\chi_2)\left|\frac{\tilde{v}}{(1+r_t^a)\tilde{a}_- + \chi_0\Upsilon(\Gamma)}\right|^{\chi_2}(1+r_t^a).$$

When $v_{i,t} = a_{i,t} - (1+r_t^a)a_{i,t-1}$ and $\tilde{v}_{i,t} := v_{i,t}/X_{t-1} = g_t\tilde{a}_{i,t} - (1+r_t^a)\tilde{a}_{i,t-1}$, we have

$$\tilde{\chi}_t(\tilde{v}_{i,t},\tilde{a}_{i,t-1};\Gamma) = \chi_t(v_{i,t},a_{i,t-1};\Gamma)/X_{t-1},$$
$$\tilde{\chi}_{1,t}(\tilde{v}_{i,t},\tilde{a}_{i,t-1};\Gamma) = \chi_{1,t}(v_{i,t},a_{i,t-1};\Gamma), \quad \text{and}$$
$$\tilde{\chi}_{2,t}(\tilde{v}_{i,t},\tilde{a}_{i,t-1};\Gamma) = \chi_{2,t}(v_{i,t},a_{i,t-1};\Gamma).$$

Workers' optimality conditions are detrended as follows.

$$\tilde{\lambda} = \tilde{c}^{-\gamma}, \tag{B.13}$$

$$\tilde{\lambda} = \beta g_t^{-\gamma}\sum_{e_1',e_2'} P(e_1',e_2'|e_1,e_2)\tilde{V}^W_{\tilde{b},t+1}(e_1',e_2',\tilde{b},\tilde{a};\Gamma) + \tilde{\varphi}^b, \tag{B.14}$$

$$\begin{aligned}\tilde{\lambda}\{1+&\eta_t\tilde{\chi}_{1,t}(g_t\tilde{a}-(1+r_t^a)\tilde{a}_-,\tilde{a}_-;\Gamma)\}\\ &= \beta g_t^{-\gamma}\sum_{e_1',e_2'} P(e_1',e_2'|e_1,e_2)\tilde{V}^W_{\tilde{a},t+1}(e_1',e_2',\tilde{b},\tilde{a};\Gamma) + \tilde{\varphi}^a,\end{aligned} \tag{B.15}$$

$$\tilde{V}^W_{\tilde{b},t}(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma) = (1-\xi)(1+r_t^b)\tilde{\lambda}, \tag{B.16}$$

$$\begin{aligned}\tilde{V}^W_{\tilde{a},t}(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma) = \tilde{\lambda}\{&(1+r_t^a) + (1+r_t^a)\eta_t\tilde{\chi}_{1,t}(g_t\tilde{a}-(1+r_t^a)\tilde{a}_-,\tilde{a}_-;\Gamma)\\ &- \eta_t\tilde{\chi}_{2,t}(g_t\tilde{a}-(1+r_t^a)\tilde{a}_-,\tilde{a}_-;\Gamma)\},\end{aligned} \tag{B.17}$$

$$\tilde{c} + g_t\tilde{b} + g_t\tilde{a} + \eta_t\tilde{\chi}_t(g_t\tilde{a}-(1+r_t^a)\tilde{a}_-,\tilde{a}_-;\Gamma) = \tilde{w}_t\Gamma e\bar{l}_t + (1-\xi)(1+r_t^b)\tilde{b}_- + (1+r_t^a)\tilde{a}_-, \tag{B.18}$$

$$\tilde{\varphi}^b \geq 0, \quad \tilde{b} \geq 0, \quad \tilde{\varphi}^b\tilde{b} = 0, \quad \text{and} \tag{B.19}$$

$$\tilde{\varphi}^a \geq 0, \quad \tilde{a} \geq 0, \quad \tilde{\varphi}^a\tilde{a} = 0. \tag{B.20}$$

The law of motion for $\Psi_t(e_1,e_2,b_-,a_-|\Gamma)$ (or, equivalently, equation (13)) is detrended to the law of motion for $\tilde{\Psi}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-|\Gamma)$ as follows.

$$\tilde{\Psi}_{t+1}(e_1',e_2',\tilde{b},\tilde{a}|\Gamma) = \int_{e_1,e_2,\tilde{b}_-,\tilde{a}_-} \big[P(e_{1,t+1} \le e_1'|e_{1,t}=e_1)\, P(e_{2,t+1} \le e_2') \\ I_{\{\tilde{b}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma)\le \tilde{b},\ \tilde{a}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma)\le \tilde{a}\}}(e_1,e_2,\tilde{b}_-,\tilde{a}_-)\big]\, d\tilde{\Psi}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-|\Gamma). \tag{B.21}$$

Entrepreneurs' optimality conditions are detrended as follows.

$$\tilde{C}_t^E + g_t\tilde{A}_t^E = \tilde{R}_t^E + (1+r_t^a)\tilde{A}_{t-1}^E, \quad t\ge 0, \quad \text{and} \tag{B.22}$$

$$(\tilde{C}_t^E)^{-\gamma} = g_t^{-\gamma}\beta_E(1+r_{t+1}^a)(\tilde{C}_{t+1}^E)^{-\gamma}, \quad t\ge 0. \tag{B.23}$$

Firms' detrended optimality conditions are the same as those derived in Online Appendix A.2. The aggregation equations (18), (19), (20), and (23) are detrended as follows.

$$\tilde{C}_t = p\tilde{C}_t^W + (1-p)\tilde{C}_t^E, \quad \tilde{C}_t^W = \int_\Gamma\int_{e_1,e_2,\tilde{b}_-,\tilde{a}_-} \tilde{c}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma)\, d\tilde{\Psi}_t dG. \tag{B.24}$$

$$\tilde{A}_t = p\tilde{A}_t^W + (1-p)\tilde{A}_t^E, \quad \tilde{A}_t^W = \int_\Gamma\int_{e_1,e_2,\tilde{b}_-,\tilde{a}_-} \tilde{a}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma)\, d\tilde{\Psi}_t dG. \tag{B.25}$$

$$\tilde{B}_t = p\tilde{B}_t^W, \quad \tilde{B}_t^W = \int_\Gamma\int_{e_1,e_2,\tilde{b}_-,\tilde{a}_-} \tilde{b}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma)\, d\tilde{\Psi}_t dG. \tag{B.26}$$

$$\tilde{\chi}_t^{agg} = p\tilde{\chi}_t^W, \ \ \tilde{\chi}_t^W = \int_\Gamma\int_{e_1,e_2,\tilde{b}_-,\tilde{a}_-} \eta_t\tilde{\chi}_t\big(g_t\tilde{a}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma) - (1+r_t^a)\tilde{a}_-,\tilde{a}_-;\Gamma\big)\, d\tilde{\Psi}_t dG. \tag{B.27}$$

Equations (12), (15), and (17) are detrended as follows.

$$\tilde{w}_t(p\bar{\Gamma}\bar{e})^{1+\omega} = \kappa L_t^\omega, \quad t\ge 0. \tag{B.28}$$

$$\tilde{F}_t = \tilde{D}_t + \tilde{B}_t, \quad t\ge 0. \tag{B.29}$$

$$(1-p)\tilde{R}_t^E = \xi(1+r_t^b)\tilde{B}_{t-1} + \tilde{\chi}_t^{agg}. \tag{B.30}$$

Equations (11) and (16) do not need to be detrended. The rest of the equilibrium conditions (*i.e.*, equations (2), (3), (6), and (8)) are detrended in Online Appendix A.2.

Given the initial conditions on $\tilde{\Psi}_0(e_1,e_2,\tilde{b}_-,\tilde{a}_-|\Gamma)$, $\tilde{A}_{-1}$, $\tilde{A}_{-1}^E$, $\tilde{K}_{-1}$, $\tilde{D}_{-1}$, $\tilde{B}_{-1}$, $\tilde{F}_{-1}$, and r_{-1} and deterministic paths of aggregate exogenous variables $\{z_t,g_t,\mu_t,\eta_t\}_{t=0}^\infty$, *(i)* individual workers' detrended policy functions $\{\tilde{c}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma),\tilde{b}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma),\tilde{a}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma)\}_{t=0}^\infty$, first-order derivatives of the detrended value functions $\{\tilde{V}_{\tilde{b},t}^W(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma),\tilde{V}_{\tilde{a},t}^W(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma)\}_{t=0}^\infty$, and detrended Lagrangian multipliers $\{\tilde{\lambda}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma),\tilde{\varphi}_t^b(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma),\tilde{\varphi}_t^a(e_1,e_2,\tilde{b}_-,\tilde{a}_-;\Gamma)\}_{t=0}^\infty$

that satisfy workers' detrended optimality conditions (B.13) - (B.20), *(ii)* conditional cumulative distributions $\{\tilde{\Psi}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-|\Gamma)\}_{t=1}^{\infty}$ that evolve over time according to equation (B.21), and *(iii)* prices and aggregate variables $\{r_t^b,r_t^a,r_t,\tilde{w}_t,\tilde{q}_t,\bar{l}_t,L_t,\tilde{\Pi}_t,\tilde{Y}_t,\tilde{I}_t,\tilde{K}_t,\tilde{F}_t,\tilde{D}_t,\tilde{TB}_t,\tilde{C}_t,\tilde{C}_t^E,\tilde{A}_t,\tilde{A}_t^E,\tilde{R}_t^E,\tilde{B}_t,\tilde{\chi}_t^{agg}\}_{t=0}^{\infty}$ satisfying entrepreneurs' detrended optimality conditions (B.22) and (B.23), firms' detrended optimality conditions (A.16) - (A.21), detrended aggregation equations (B.24) - (B.27), and other detrended equilibrium conditions (A.22), (A.23), (A.25), (A.27), (11), (B.28), (B.29), (16), and (B.30) constitute the detrended equilibrium of the economy.

Since the resource constraint (A.11) holds in the undetrended HASOE equilibrium, the detrended resource consstraint (A.26) also holds in the detrended HASOE equilibrium.

B.3 Workers' Detrended and Normalized Optimality Conditions

Workers' detrended optimality conditions (B.13) - (B.20) can be normalized such that the predictable component of idiosyncratic productivity Γ is irrelevant under the normalized conditions. Let

$$\hat{w}_t := \tilde{w}_t/\tilde{w}_{ss}, \quad \hat{e}_{i,t} := e_{i,t}/\bar{e}, \quad \hat{l}_t := \bar{l}_t/\bar{l}_{ss},$$

$$\bar{e}_1 := E[e_{1,i,t}], \quad \hat{e}_{1,i,t} := e_{1,i,t}/\bar{e}_1, \quad \bar{e}_2 := E[e_{2,i,t}], \quad \hat{e}_{2,i,t} := e_{2,i,t}/\bar{e}_2,$$

$$\hat{c}_{i,t} := \tilde{c}_{i,t}/\Upsilon(\Gamma), \quad \hat{b}_{i,t} := \tilde{b}_{i,t}/\Upsilon(\Gamma), \quad \hat{a}_{i,t} := \tilde{a}_{i,t}/\Upsilon(\Gamma),$$

$$\hat{\lambda}_{i,t} := \tilde{\lambda}_{i,t}/\Upsilon(\Gamma)^{-\gamma}, \quad \hat{\varphi}_{i,t}^b := \tilde{\varphi}_{i,t}^b/\Upsilon(\Gamma)^{-\gamma}, \quad \hat{\varphi}_{i,t}^a := \tilde{\varphi}_{i,t}^a/\Upsilon(\Gamma)^{-\gamma},$$

$$\hat{C}_t^W := \tilde{C}_t^W/\Upsilon(\bar{\Gamma}), \quad \hat{B}_t^W := \tilde{B}_t^W/\Upsilon(\bar{\Gamma}), \quad \hat{A}_t^W := \tilde{A}_t^W/\Upsilon(\bar{\Gamma}), \quad \hat{\chi}_t^W := \tilde{\chi}_t^W/\Upsilon(\bar{\Gamma}),$$

$$\hat{\Psi}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) := \tilde{\Psi}_t(\bar{e}_1\hat{e}_1,\bar{e}_2\hat{e}_2,\Upsilon(\Gamma)\hat{b}_-,\Upsilon(\Gamma)\hat{a}_-|\Gamma),$$

$$\hat{V}_{\hat{b},t}^W(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) := \tilde{V}_{\tilde{b},t}^W(\bar{e}_1\hat{e}_1,\bar{e}_2\hat{e}_2,\Upsilon(\Gamma)\hat{b}_-,\Upsilon(\Gamma)\hat{a}_-;\Gamma) \;/\; \Upsilon(\Gamma)^{-\gamma},$$

$$\hat{V}_{\hat{a},t}^W(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) := \tilde{V}_{\tilde{a},t}^W(\bar{e}_1\hat{e}_1,\bar{e}_2\hat{e}_2,\Upsilon(\Gamma)\hat{b}_-,\Upsilon(\Gamma)\hat{a}_-;\Gamma) \;/\; \Upsilon(\Gamma)^{-\gamma},$$

$$\hat{c}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) := \tilde{c}_t(\bar{e}_1\hat{e}_1,\bar{e}_2\hat{e}_2,\Upsilon(\Gamma)\hat{b}_-,\Upsilon(\Gamma)\hat{a}_-;\Gamma)/\Upsilon(\Gamma),$$

$$\hat{b}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) := \tilde{b}_t(\bar{e}_1\hat{e}_1,\bar{e}_2\hat{e}_2,\Upsilon(\Gamma)\hat{b}_-,\Upsilon(\Gamma)\hat{a}_-;\Gamma)/\Upsilon(\Gamma),$$

$$\hat{a}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) := \tilde{a}_t(\bar{e}_1\hat{e}_1,\bar{e}_2\hat{e}_2,\Upsilon(\Gamma)\hat{b}_-,\Upsilon(\Gamma)\hat{a}_-;\Gamma)/\Upsilon(\Gamma),$$

$$\hat{\lambda}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) := \tilde{\lambda}_t(\bar{e}_1\hat{e}_1,\bar{e}_2\hat{e}_2,\Upsilon(\Gamma)\hat{b}_-,\Upsilon(\Gamma)\hat{a}_-;\Gamma)/\Upsilon(\Gamma)^{-\gamma},$$

$$\hat{\varphi}_t^b(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) := \tilde{\varphi}_t^b(\bar{e}_1\hat{e}_1,\bar{e}_2\hat{e}_2,\Upsilon(\Gamma)\hat{b}_-,\Upsilon(\Gamma)\hat{a}_-;\Gamma)/\Upsilon(\Gamma)^{-\gamma},$$

$$\hat{\varphi}_t^a(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) := \tilde{\varphi}_t^a(\bar{e}_1\hat{e}_1,\bar{e}_2\hat{e}_2,\Upsilon(\Gamma)\hat{b}_-,\Upsilon(\Gamma)\hat{a}_-;\Gamma)/\Upsilon(\Gamma)^{-\gamma}, \quad \text{and}$$

$$\hat{\chi}_t(\hat{v},\hat{a}_-) := \chi_1\left|\frac{\hat{v}}{(1+r_t^a)\hat{a}_-+\chi_0}\right|^{\chi_2}\left((1+r_t^a)\hat{a}_-+\chi_0\right).$$

(In the first line of the equations, $\tilde{w}_{ss}$ and $\bar{l}_{ss}$ are the steady state values of $\tilde{w}_t$ and $\bar{l}_t$, respectively.)

The first-order derivatives of $\hat{\chi}_t(\hat{v},\hat{a}_-)$ are

$$\hat{\chi}_{1,t}(\hat{v},\hat{a}_-) = sign(\hat{v})\chi_1\chi_2\left|\frac{\hat{v}}{(1+r_t^a)\hat{a}_- + \chi_0}\right|^{\chi_2-1}, \quad \text{and}$$
$$\hat{\chi}_{2,t}(\hat{v},\hat{a}_-) = \chi_1(1-\chi_2)\left|\frac{\hat{v}}{(1+r_t^a)\hat{a}_- + \chi_0}\right|^{\chi_2}(1+r_t^a).$$

When $\tilde{v}_{i,t} = g_t\tilde{a}_{i,t} - (1+r_t^a)\tilde{a}_{i,t-1}$ and $\hat{v}_{i,t} := \tilde{v}_{i,t}/\Upsilon(\Gamma) = g_t\hat{a}_{i,t} - (1+r_t^a)\hat{a}_{i,t-1}$, we have

$$\hat{\chi}_t(\hat{v}_{i,t},\hat{a}_{i,t-1}) = \tilde{\chi}_t(\tilde{v}_{i,t},\tilde{a}_{i,t-1};\Gamma)/\Upsilon(\Gamma),$$
$$\hat{\chi}_{1,t}(\hat{v}_{i,t},\hat{a}_{i,t-1}) = \tilde{\chi}_{1,t}(\tilde{v}_{i,t},\tilde{a}_{i,t-1};\Gamma), \quad \text{and}$$
$$\hat{\chi}_{2,t}(\hat{v}_{i,t},\hat{a}_{i,t-1}) = \tilde{\chi}_{2,t}(\tilde{v}_{i,t},\tilde{a}_{i,t-1};\Gamma).$$

Workers' detrended optimality conditions (B.13) - (B.20) are normalized as follows.

$$\hat{\lambda} = \hat{c}^{-\gamma}, \tag{B.31}$$

$$\hat{\lambda} = \beta g_t^{-\gamma}\sum_{\hat{e}_1',\hat{e}_2'} P(\hat{e}_1',\hat{e}_2'|\hat{e}_1,\hat{e}_2)\hat{V}^W_{\hat{b},t+1}(\hat{e}_1',\hat{e}_2',\hat{b},\hat{a}) + \hat{\varphi}^b, \tag{B.32}$$

$$\begin{aligned}\hat{\lambda}\{1+&\eta_t\hat{\chi}_{1,t}(g_t\hat{a}-(1+r_t^a)\hat{a}_-,\hat{a}_-)\}\\ &= \beta g_t^{-\gamma}\sum_{\hat{e}_1',\hat{e}_2'} P(\hat{e}_1',\hat{e}_2'|\hat{e}_1,\hat{e}_2)\hat{V}^W_{\hat{a},t+1}(\hat{e}_1',\hat{e}_2',\hat{b},\hat{a}) + \hat{\varphi}^a,\end{aligned} \tag{B.33}$$

$$\hat{V}^W_{\hat{b},t}(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) = (1-\xi)(1+r_t^b)\hat{\lambda}, \tag{B.34}$$

$$\begin{aligned}\hat{V}^W_{\hat{a},t}(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) = \hat{\lambda}\{&(1+r_t^a) + (1+r_t^a)\eta_t\hat{\chi}_{1,t}(g_t\hat{a}-(1+r_t^a)\hat{a}_-,\hat{a}_-)\\ &-\eta_t\hat{\chi}_{2,t}(g_t\hat{a}-(1+r_t^a)\hat{a}_-,\hat{a}_-)\},\end{aligned} \tag{B.35}$$

$$\hat{c} + g_t\hat{b} + g_t\hat{a} + \eta_t\hat{\chi}_t(g_t\hat{a}-(1+r_t^a)\hat{a}_-,\hat{a}_-) = \hat{w}_t\hat{e}\hat{l}_t + (1-\xi)(1+r_t^b)\hat{b}_- + (1+r_t^a)\hat{a}_-, \tag{B.36}$$

$$\hat{\varphi}^b \geq 0, \quad \hat{b} \geq 0, \quad \hat{\varphi}^b\hat{b} = 0, \quad \text{and} \tag{B.37}$$

$$\hat{\varphi}^a \geq 0, \quad \hat{a} \geq 0, \quad \hat{\varphi}^a\hat{a} = 0. \tag{B.38}$$

Note that Γ does not appear in the normalized equations (B.31) - (B.38).

The law of motion for $\tilde{\Psi}_t(e_1,e_2,\tilde{b}_-,\tilde{a}_-|\Gamma)$ (or, equivalently, equation (B.21)) is normalized to the law of motion for $\hat{\Psi}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)$ as follows.

$$\begin{aligned}\hat{\Psi}_{t+1}(\hat{e}_1',\hat{e}_2',\hat{b},\hat{a}) = \int_{\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-} &\big[P(\hat{e}_{1,t+1}\leq \hat{e}_1'|\hat{e}_{1,t}=\hat{e}_1)\,P(\hat{e}_{2,t+1}\leq \hat{e}_2') \\ &I_{\{\hat{b}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\leq \hat{b},\ \hat{a}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\leq \hat{a}\}}(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\big]\,d\hat{\Psi}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-).\end{aligned} \tag{B.39}$$

Using the normalized variables and functions, the detreded aggregation equations (B.24) - (B.27) can be rewritten as follows.

$$\tilde{C}_t = p\tilde{C}_t^W + (1-p)\tilde{C}_t^E, \quad \tilde{C}_t^W = \Upsilon(\bar{\Gamma})\hat{C}_t^W, \quad \hat{C}_t^W = \int_{\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-} \hat{c}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\,d\hat{\Psi}_t. \tag{B.40}$$

$$\tilde{A}_t = p\tilde{A}_t^W + (1-p)\tilde{A}_t^E, \quad \tilde{A}_t^W = \Upsilon(\bar{\Gamma})\hat{A}_t^W, \quad \hat{A}_t^W = \int_{\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-} \hat{a}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\,d\hat{\Psi}_t. \tag{B.41}$$

$$\tilde{B}_t = p\tilde{B}_t^W, \quad \tilde{B}_t^W = \Upsilon(\bar{\Gamma})\hat{B}_t^W, \quad \hat{B}_t^W = \int_{\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-} \hat{b}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\,d\hat{\Psi}_t. \tag{B.42}$$

$$\tilde{\chi}_t^{agg} = p\tilde{\chi}_t^W, \ \ \tilde{\chi}_t^W = \Upsilon(\bar{\Gamma})\hat{\chi}_t^W, \ \ \hat{\chi}_t^W = \int_{\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-} \eta_t\hat{\chi}_t\big(g_t\hat{a}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)-(1+r_t^a)\hat{a}_-,\hat{a}_-\big)\,d\hat{\Psi}_t. \tag{B.43}$$

Using the normalized variables, functions, and equations, the detrended equilibrium of the HASOE economy in Online Appendix B.2 can be rewritten as follows.

Given the initial conditions on $\hat{\Psi}_0(e_1,e_2,\tilde{b}_-,\tilde{a}_-)$, $\tilde{A}_{-1}$, $\tilde{A}_{-1}^E$, $\tilde{K}_{-1}$, $\tilde{D}_{-1}$, $\tilde{B}_{-1}$, $\tilde{F}_{-1}$, and r_{-1} and deterministic paths of aggregate exogenous variables $\{z_t,g_t,\mu_t,\eta_t\}_{t=0}^{\infty}$, *(i)* individual workers' detrended and normalized policy functions $\{\hat{c}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-),\hat{b}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-),\hat{a}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\}_{t=0}^{\infty}$, first-order derivatives of the detrended and normalized value functions $\{\hat{V}_{\hat{b},t}^W(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)$, $\hat{V}_{\hat{a},t}^W(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\}_{t=0}^{\infty}$, and detrended and normalized Lagrangian multipliers $\{\hat{\lambda}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)$, $\hat{\varphi}_t^b(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-),\hat{\varphi}_t^a(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\}_{t=0}^{\infty}$ that satisfy workers' detrended and normalized optimality conditions (B.31) - (B.38), *(ii)* cumulative distributions $\{\hat{\Psi}_t(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\}_{t=1}^{\infty}$ that evolve over time according to equation (B.39), and *(iii)* prices and aggregate variables $\{r_t^b,r_t^a,r_t,\tilde{w}_t,\tilde{q}_t,\bar{l}_t,L_t,\tilde{\Pi}_t$, $\tilde{Y}_t,\tilde{I}_t,\tilde{K}_t,\tilde{F}_t,\tilde{D}_t,\tilde{TB}_t,\tilde{C}_t,\tilde{C}_t^E,\tilde{A}_t,\tilde{A}_t^E,\tilde{R}_t^E,\tilde{B}_t,\tilde{\chi}_t^{agg}\}_{t=0}^{\infty}$ satisfying entrepreneurs' detrended optimality conditions (B.22) and (B.23), firms' detrended optimality conditions (A.16) - (A.21), detrended aggregation equations (B.40) - (B.43), and other detrended equilibrium conditions (A.22), (A.23), (A.25), (A.27), (11), (B.28), (B.29), (16), and (B.30) constitute the detrended equilibrium of the economy.

When I numerically solve workers' problem, I use their detrended and normalized optimality conditions (B.31) - (B.38).

B.4 Steady State of the Detrended Equilibrium

For any variable x_t and any function $f_t(\cdot)$, let x_{ss} and $f_{ss}(\cdot)$ denote their steady state values, respectively. In the steady state of the detrended equilibrium, the following equations hold.

$$z_{ss} = 1, \quad g_{ss} = g^*, \quad \mu_{ss} = 1, \quad \text{and} \quad \eta_{ss} = 1.$$
$$\tilde{D}_{ss} = \tilde{D}^* \quad \text{and} \quad \tilde{Y}_{ss} = \tilde{Y}^* \quad \text{(by definition).}$$

In the steady state, workers' detrended and normalized policy functions $\hat{c}_{ss}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)$, $\hat{b}_{ss}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)$, and $\hat{a}_{ss}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)$, first-order derivatives of the detrended and normalized value functions $\hat{V}^W_{\hat{b},ss}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)$ and $\hat{V}^W_{\hat{a},ss}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)$, and detrended and normalized Lagrangian multipliers $\hat{\lambda}_{ss}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)$, $\hat{\varphi}^b_{ss}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)$, and $\hat{\varphi}^a_{ss}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)$ solve the following optimality conditions.

$$\hat{\lambda} = \hat{c}^{-\gamma}, \tag{B.44}$$

$$\hat{\lambda} = \beta g_{ss}^{-\gamma} \sum_{\hat{e}'_1, \hat{e}'_2} P(\hat{e}'_1, \hat{e}'_2 | \hat{e}_1, \hat{e}_2) \hat{V}^W_{\hat{b},ss}(\hat{e}'_1, \hat{e}'_2, \hat{b}, \hat{a}) + \hat{\varphi}^b, \tag{B.45}$$

$$\begin{aligned} &\hat{\lambda}\{1+\hat{\chi}_{1,ss}(g_{ss}\hat{a} - (1+r^a_{ss})\hat{a}_-, \hat{a}_-)\} \\ &\quad = \beta g_{ss}^{-\gamma} \sum_{\hat{e}'_1, \hat{e}'_2} P(\hat{e}'_1, \hat{e}'_2 | \hat{e}_1, \hat{e}_2) \hat{V}^W_{\hat{a},ss}(\hat{e}'_1, \hat{e}'_2, \hat{b}, \hat{a}) + \hat{\varphi}^a, \end{aligned} \tag{B.46}$$

$$\hat{V}^W_{\hat{b},ss}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) = (1-\xi)(1+r^b_{ss})\hat{\lambda}, \tag{B.47}$$

$$\begin{aligned} \hat{V}^W_{\hat{a},ss}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-) = \hat{\lambda}\{&(1+r^a_{ss}) + (1+r^a_{ss})\hat{\chi}_{1,ss}(g_{ss}\hat{a} - (1+r^a_{ss})\hat{a}_-, \hat{a}_-) \\ &- \hat{\chi}_{2,ss}(g_{ss}\hat{a} - (1+r^a_{ss})\hat{a}_-, \hat{a}_-)\}, \end{aligned} \tag{B.48}$$

$$\hat{c} + g_{ss}\hat{b} + g_{ss}\hat{a} + \hat{\chi}_{ss}(g_{ss}\hat{a} - (1+r^a_{ss})\hat{a}_-, \hat{a}_-) = \hat{w}_{ss}\hat{e}\hat{l}_{ss} + (1-\xi)(1+r^b_{ss})\hat{b}_- + (1+r^a_{ss})\hat{a}_-, \tag{B.49}$$

$$\hat{\varphi}^b \geq 0, \quad \hat{b} \geq 0, \quad \hat{\varphi}^b \hat{b} = 0, \quad \text{and} \tag{B.50}$$

$$\hat{\varphi}^a \geq 0, \quad \hat{a} \geq 0, \quad \hat{\varphi}^a \hat{a} = 0. \tag{B.51}$$

In the steady state, the evolution equation for $\hat{\Psi}_t(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}-)$ becomes

$$\begin{aligned} \hat{\Psi}_{ss}(\hat{e}'_1, \hat{e}'_2, \hat{b}, \hat{a}) = \int_{\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-} &\big[P(\hat{e}_{1,t+1} \leq \hat{e}'_1 | \hat{e}_{1,t} = \hat{e}_1)\, P(\hat{e}_{2,t+1} \leq \hat{e}'_2) \\ &I_{\{\hat{b}_{ss}(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) \leq \hat{b},\ \hat{a}_{ss}(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-) \leq \hat{a}\}}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-)\big]\, d\hat{\Psi}_{ss}(\hat{e}_1, \hat{e}_2, \hat{b}_-, \hat{a}_-). \end{aligned} \tag{B.52}$$

In the steady state, entrepreneurs' detrended optimality conditions become

$$\tilde{C}^E_{ss} = \tilde{R}^E_{ss} + (1+r^a_{ss} - g_{ss})\tilde{A}^E_{ss}, \quad \text{and} \tag{B.53}$$

$$g_{ss}^{-\gamma}\beta_E(1+r_{ss}^a)=1. \tag{B.54}$$

In the steady state, firms' detrended optimality conditions become (A.32) - (A.37).

In the steady state, the detrended aggregation equations become

$$\tilde{C}_{ss}=p\Upsilon(\bar{\Gamma})\hat{C}_{ss}^W+(1-p)\tilde{C}_{ss}^E,\quad \hat{C}_{ss}^W=\int_{\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-}\hat{c}_{ss}(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\,d\hat{\Psi}_{ss}, \tag{B.55}$$

$$\tilde{A}_{ss}=p\Upsilon(\bar{\Gamma})\hat{A}_{ss}^W+(1-p)\tilde{A}_{ss}^E,\quad \hat{A}_{ss}^W=\int_{\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-}\hat{a}_{ss}(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\,d\hat{\Psi}_{ss}, \tag{B.56}$$

$$\tilde{B}_{ss}=p\Upsilon(\bar{\Gamma})\hat{B}_{ss}^W,\quad \hat{B}_{ss}^W=\int_{\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-}\hat{b}_{ss}(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)\,d\hat{\Psi}_{ss},\quad \text{and} \tag{B.57}$$

$$\tilde{\chi}_{ss}^{agg}=p\Upsilon(\bar{\Gamma})\hat{\chi}_{ss}^W,\ \ \hat{\chi}_{ss}^W=\int_{\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-}\hat{\chi}_{ss}\big(g_{ss}\hat{a}_{ss}(\hat{e}_1,\hat{e}_2,\hat{b}_-,\hat{a}_-)-(1+r_{ss}^a)\hat{a}_-,\hat{a}_-\big)\,d\hat{\Psi}_{ss}. \tag{B.58}$$

In the steady state, equilibrium conditions (A.22), (A.23), (A.25), and (A.27) become (A.38), (A.39), (A.41), and (A.43), respectively, and equilibrium conditions (11), (B.28), (B.29), (16), and (B.30) become

$$L_{ss}=p\bar{\Gamma}\bar{e}\bar{l}_{ss}, \tag{B.59}$$

$$\tilde{w}_{ss}(p\bar{\Gamma}\bar{e})^{1+\omega}=\kappa L^{\omega}, \tag{B.60}$$

$$\tilde{F}_{ss}=\tilde{D}_{ss}+\tilde{B}_{ss}, \tag{B.61}$$

$$r_{ss}^b=r_{ss},\quad \text{and} \tag{B.62}$$

$$(1-p)\tilde{R}_{ss}^E=\xi(1+r_{ss}^b)\tilde{B}_{ss}+\tilde{\chi}_{ss}^{agg}. \tag{B.63}$$

Using equations (A.38), (A.39), (A.32), (A.37), (A.33), (A.35),(A.34), (B.61), and (A.36), we can derive a relationship among stock variables $\tilde{A}_{ss}$, $\tilde{B}_{ss}$, $\tilde{K}_{ss}$, and $\tilde{D}_{ss}$ as follows.

$$\begin{aligned}(1+r_{ss}^a-g_{ss})\tilde{A}_{ss}&=(1+r_{ss}^a-g_{ss})\tilde{q}_{ss}=\tilde{\Pi}_{ss}=\tilde{Y}_{ss}-\tilde{w}_{ss}L_{ss}-\tilde{I}_{ss}-(1+r_{ss}-g_{ss})\tilde{F}_{ss}\\&=\alpha\tilde{Y}_{ss}-\tilde{I}_{ss}-(1+r_{ss}-g_{ss})\tilde{F}_{ss}=(r_{ss}+\delta)\tilde{K}_{ss}-\tilde{I}_{ss}-(1+r_{ss}-g_{ss})\tilde{F}_{ss}\\&=(r_{ss}+\delta)\tilde{K}_{ss}-(g_{ss}-1+\delta)\tilde{K}_{ss}-(1+r_{ss}-g_{ss})\tilde{F}_{ss}\\&=(r_{ss}+\delta)\tilde{K}_{ss}-(g_{ss}-1+\delta)\tilde{K}_{ss}-(1+r_{ss}-g_{ss})(\tilde{D}_{ss}+\tilde{B}_{ss})\\&=(1+r_{ss}-g_{ss})(\tilde{K}_{ss}-\tilde{D}_{ss}-\tilde{B}_{ss})=(1+r_{ss}^a-g_{ss})(\tilde{K}_{ss}-\tilde{D}_{ss}-\tilde{B}_{ss}).\end{aligned}$$

$$\therefore \tilde{A}_{ss}+\tilde{B}_{ss}=\tilde{K}_{ss}-\tilde{D}_{ss}. \tag{B.64}$$

C Solution Method

I solve the detrended equilibrium of the RASOE and HASOE models (characterized in Online Appendices A.2 and B.2, respectively) using Auclert et al. (2021)'s method. In this section, I discuss how this method is applied to solve the models.

C.1 Steady State

The first step is to solve the steady state. The steady state of the (detrended) RASOE economy is straightforward to compute using equations (A.28) - (A.43). To compute the steady state of the (detrended) HASOE economy, I solve workers' (detrended and normalized) policy functions and stationary distribution that satisfy equations (B.44) - (B.52). Auclert et al. (2021) develop a fast algorithm that extends Carroll (2006)'s method of endogenous gridpoints to a two-asset environment in order to solve the steady state of their two-asset HANK model. Since the worker block of my model is almost identical to the household block of their model, I closely follow this algorithm to solve the workers' problem in the steady state. See Appendix E.1 of Auclert et al. (2021) for details of this algorithm.[3]

C.2 Sequence Space Approach

Once the steady state is pinned down, Auclert et al. (2021)'s method computes the Jacobians of 'blocks'. Here, a 'block' is a function that maps the sequences of input variables $\{x_{1,t},\cdots,x_{n_x,t}\}_{t=0}^{\infty}$ into the sequences of output variables $\{y_{1,t},\cdots,y_{n_y,t}\}_{t=0}^{\infty}$ using a subset of equilibrium conditions. The Jacobian of each block is a matrix composed of $(\partial y_{j,s}/\partial x_{i,t})_{1\leq i\leq n_x,\ 1\leq j\leq n_y,\ s,t\geq 0}$. For example, the worker block in my HASOE model maps the sequences of input variables $\{\tilde{w}_t, r_t^a, r_t^b, g_t, \bar{l}_t, \eta_t\}_{t=0}^{\infty}$ into the sequences of output variables $\{\tilde{C}_t^W, \tilde{B}_t^W, \tilde{A}_t^W, \tilde{\chi}_t^W\}_{t=0}^{\infty}$ using equilibrium conditions (B.31) - (B.43).[4] The Jacobian of the worker block is composed of $(\partial y_s/\partial x_t)_{x\in\{\tilde{w},r^a,r^b,g,\bar{l},\eta\},\ y\in\{\tilde{C}^W,\tilde{B}^W,\tilde{A}^W,\tilde{\chi}^W\},\ s,t\geq 0}$.

Figure C.1 is a directed acyclical graph (DAG) representation of the detrended equilibrium of the HASOE model, in which blocks, input variables, and output variables are indicated. Both the blue rectangles and red ellipses represent blocks of the equilibrium. For each block, variables coming into the block and variables coming out of the block (indicated by arrows connecting

[3]For grids, I use 10 gridpoints for $\hat{e}_1$ and $\hat{e}_2$, respectively, 50 gridpoints for $\hat{b}_-$, and 70 gridpoints for $\hat{a}_-$.

[4]Since I use normalized optimality conditions (B.31) - (B.38) to solve workers' problem, I first compute the Jacobian for normalized variables and then scale it to the one for unnormalized variables. For example, to compute $(\partial \tilde{C}_s^W/\partial \tilde{w}_t)$, I first compute $(\partial \hat{C}_s^W/\partial \hat{w}_t)$, and then scale it using the following equation:

$$\frac{\partial \tilde{C}_s^W}{\partial \tilde{w}_t} = \frac{\partial\left(\Upsilon(\bar{\Gamma})\hat{C}_s^W\right)}{\partial\left(\tilde{w}_{ss}\hat{w}_t\right)} = \frac{\Upsilon(\bar{\Gamma})}{\tilde{w}_{ss}}\frac{\partial \hat{C}_s^W}{\partial \hat{w}_{ss}}.$$

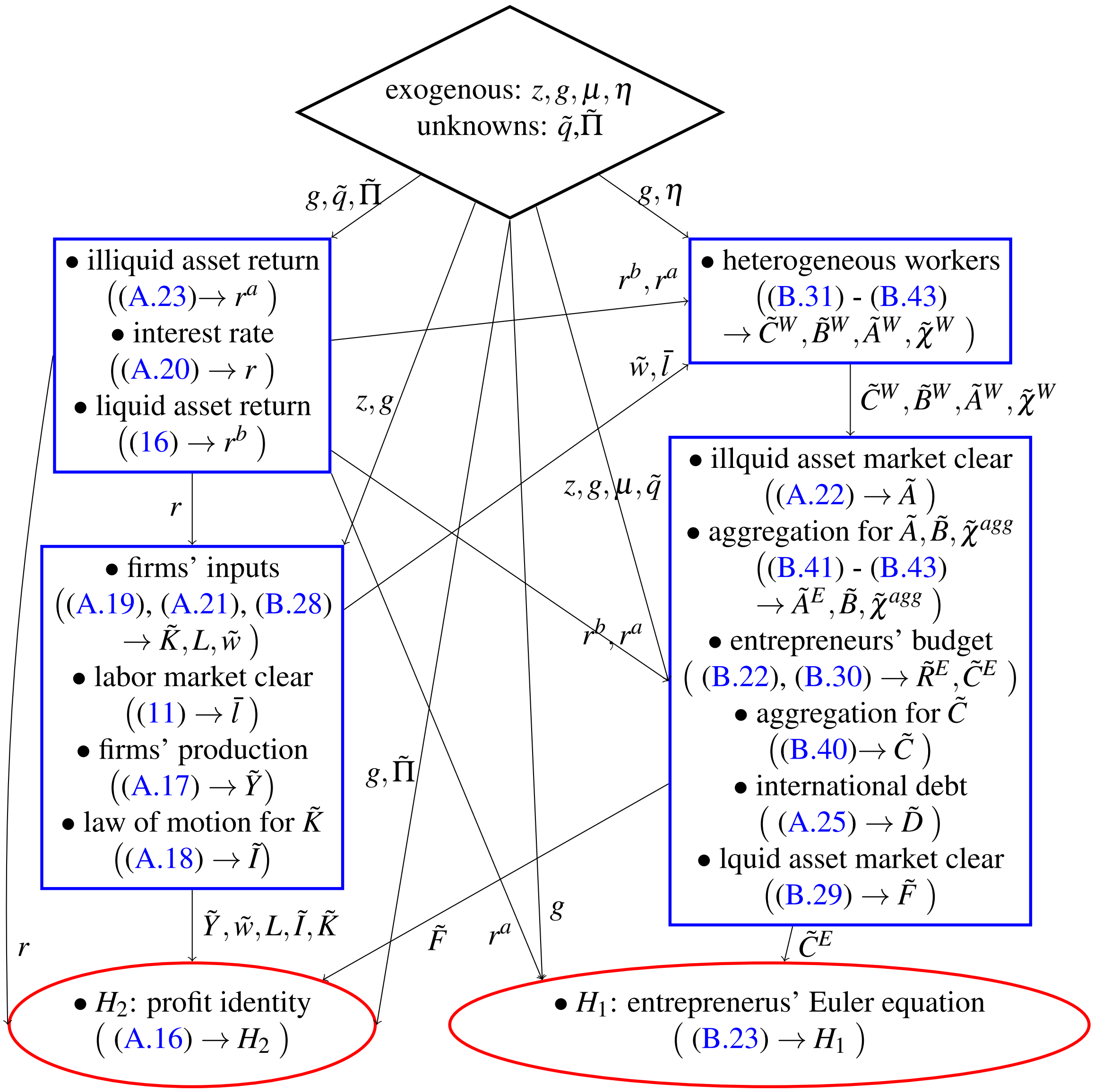


Figure C.1: DAG Representation of the Detrended Equilibrium of the HASOE Model

blocks) are input and output variables of the block, respectively. Within each block, the bullet points and following parentheses indicate the names of the equilibrium conditions, corresponding equation numbers, and output variables pinned down by the equilibrium conditions.

Following Auclert et al. (2021)'s notations, let Z denote a stacked vector of the sequences of exogenous variables and U be a stacked vector of the sequences of unknown variables (indicated in the black diamond box in Figure C.1). Moreover, let $H(U,Z)$ be a function that maps U and Z into a stacked vector of the sequences of target variables $\{H_{1,t}, H_{2,t}\}_{t=0}^{\infty}$, which are the output variables of the red ellipses in Figure C.1 and are defined as

$$H_{1,t} = g_t^{-\gamma}\beta_E(1+r_{t+1}^a)(\tilde{C}_{t+1}^E)^{-\gamma} - (\tilde{C}_t^E)^{-\gamma}, \quad \text{and}$$
$$H_{2,t} = \tilde{Y}_t - \tilde{w}_t L_t - \tilde{I}_t - \frac{\phi}{2}\left(\frac{\tilde{K}_t}{\tilde{K}_{t-1}}g_t - g^*\right)^2 \tilde{K}_{t-1} + g_t\tilde{F}_t - (1+r_{t-1})\tilde{F}_{t-1} - \tilde{\Pi}_t.$$

Under this formulation, 'solving the model' boils down to finding U that satisfies

$$H(U,Z) = 0$$

for a given Z. Under the first-order approximation regarding U and Z, this equation becomes

$$\begin{aligned} &H_U dU + H_Z dZ = 0 \\ \Leftrightarrow &dU = -H_U^{-1} H_Z dZ. \end{aligned} \tag{C.1}$$

By combining the Jacobians of the blocks through the Chain Rule, Auclert et al. (2021)'s method computes H_U and H_Z. Then, the method solves dU using equation (C.1) and recovers the linearized dynamics of other variables by combining the Jacobians again along the directed acyclical graph in Figure C.1.

In the whole procedure of solving the HASOE model, i) computing the steady state and ii) computing the Jacobian of the heterogeneous worker block are the most time-consuming steps. In particular, calibrating β, χ_1, and χ_2 requires solving the steady state multiple times, and this step takes longer than a day. However, once these parameters are calibrated and I have both the computed steady state and Jacobian of the worker block in my hand, the rest of the steps required to solve the model are very fast. This is why Bayesian estimation of the model is possible as long as the parameters to be estimated do not affect the steady state and the Jacobian of the worker block.

As Auclert et al. (2021) notes, this method can also be used to solve representative-agent models. In this paper, I also solve my RASOE model using this method. Figure C.2 presents a DAG representation of the detrended equilibrium of the RASOE model.

In implementation of the sequence space approach, sequences of the equilibrium variables over an infinite horizon must be truncated to a finite horizon. In this paper, I truncate sequences at $T = 700$ when solving models and drop the last seven periods further when evaluating moments. In Online Appendix E, I verify that under this implementation, the truncation errors barely affect model statistics at the posterior mode.

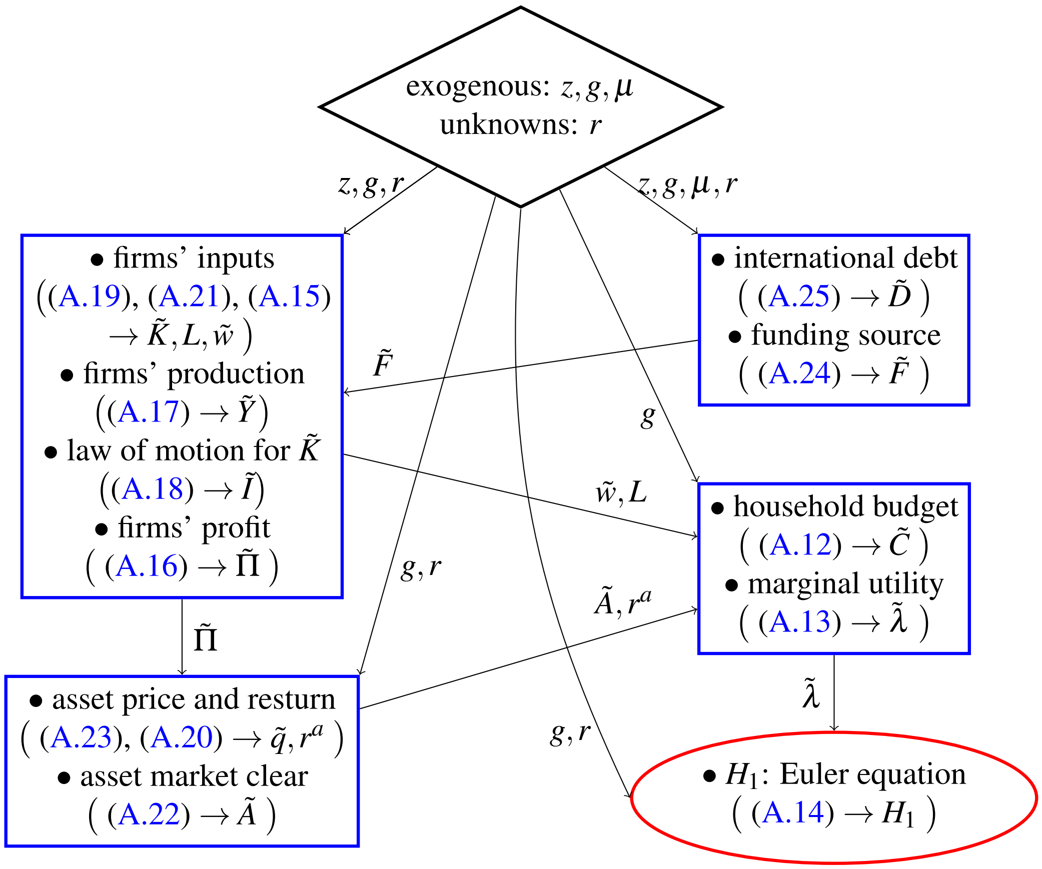


Figure C.2: DAG Representation of the Detrended Equilibrium of the RASOE Model

D Recovering the Original Equilibrium

Once the detrended equilibrium is solved, we can recover the original equilibrium. There are three types of variables in the original equilibrium that I must recover: i) observable variables ($\Delta \log Y_t$, $\Delta \log C_t$, $\Delta \log I_t$, and $\Delta(TB_t/Y_t)$), which do not exhibit a stochastic trend, ii) flow variables, which exhibit a stochastic trend and are detrended by X_{t-1} in the detrended equilibrium, and iii) stock variables, which exhibit a stochastic trend and are detrended by X_t in the detrended equilibrium. In this section, I discuss how I recover their impulse responses.

D.1 Observable Variables ($\Delta \log Y_t$, $\Delta \log C_t$, $\Delta \log I_t$, and $\Delta(TB_t/Y_t)$)

The observable variables in the original equilibrium can be described with variables in the detrended equilibrium as follows.

$$\begin{aligned} \Delta \log Y_t &= \Delta \log \tilde{Y}_t - \Delta \log \tilde{Y}_{t-1} + \log g_{t-1}, \\ \Delta \log C_t &= \Delta \log \tilde{C}_t - \Delta \log \tilde{C}_{t-1} + \log g_{t-1}, \\ \Delta \log I_t &= \Delta \log \tilde{I}_t - \Delta \log \tilde{I}_{t-1} + \log g_{t-1}, \quad \text{and} \\ \Delta(TB_t/Y_t) &= (\tilde{TB}_t/\tilde{Y}_t) - (\tilde{TB}_{t-1}/\tilde{Y}_{t-1}). \end{aligned} \tag{D.1}$$

For the purpose of this paper, I must compute the impulse responses of observable variables in terms of their *level* deviation. By solving the detrended equilibrium using Auclert et al. (2021)'s method, for any variable $\tilde{M}_t$ in the detrended equilibrium, I obtain $d(\tilde{M}_t) := \tilde{M}_t - \tilde{M}_{ss}$, where $\tilde{M}_{ss}$ is the steady state value of $\tilde{M}_t$. Using the relationships described in equation (D.1), I compute the observable variables' impulse responses as follows.

$$\begin{aligned} IRF^{level}_{\Delta \log Y}(t) &= d(\Delta \log \tilde{Y}_t) - d(\Delta \log \tilde{Y}_{t-1}) + d(\log g_{t-1}), \\ IRF^{level}_{\Delta \log C}(t) &= d(\Delta \log \tilde{C}_t) - d(\Delta \log \tilde{C}_{t-1}) + d(\log g_{t-1}), \\ IRF^{level}_{\Delta \log I}(t) &= d(\Delta \log \tilde{I}_t) - d(\Delta \log \tilde{I}_{t-1}) + d(\log g_{t-1}), \quad \text{and} \\ IRF^{level}_{\Delta(TB/Y)}(t) &= d\left(\frac{\tilde{TB}_t}{\tilde{Y}_t}\right) - d\left(\frac{\tilde{TB}_{t-1}}{\tilde{Y}_{t-1}}\right). \end{aligned}$$

D.2 Flow Variables

Flow variables in the original equilibrium, such as Y_t, C_t, I_t, Π_t, and w_t, exhibit a stochastic trend and are detrended by X_{t-1} in the detrended equilibrium. For the purpose of this paper, I must compute the impulse responses of the flow variables in terms of their *ratio* deviation from the balanced growth path. To this end, I first define the 'constant growth trend of the balanced growth path' X_t^* as follows. Given that a shock hits the economy at period 0,

$$X_t^* := (g^*)^{t+1} X_{-1}.$$

Let M_t^f denote one of the flow variables in the original equilibrium, $\tilde{M}_t^f := M_t^f / X_{t-1}$ denote its detrended variable, and $\tilde{M}_{ss}^f$ denote the steady state value of $\tilde{M}_t^f$ in the detrended equilibrium. M_t^f on the balanced growth path, which I denote as M_t^{f*}, is determined by

$$M_t^{f*} = \tilde{M}_{ss}^f X_{t-1}^*, \quad t \geq 0.$$

This is the path of M_t^f when there is no shock. The impulse response of M_t^f in terms of their ratio deviation from the balanced growth path can now be written as

$$IRF_{M^f}^{ratio}(t) = \frac{M_t^f - M_t^{f*}}{M_t^{f*}} = \frac{M_t^f/X_{t-1}^* - \tilde{M}_{ss}^f}{\tilde{M}_{ss}^f}.$$

I compute this impulse response as follows. By solving the detrended equilibrium using Auclert et al. (2021)'s method, I obtain $d\tilde{M}_t^f = \tilde{M}_t^f - \tilde{M}_{ss}^f$, where the d-operator on the left-hand side means the level deviation from the steady state in the detrended equilibrium (as defined in Online Appendix D.1). Then, I use the following equation, which holds under the first-order approximation, to obtain $d(M_t^f/X_{t-1}^*) = M_t^f/X_{t-1}^* - \tilde{M}_{ss}^f$.

$$\begin{aligned} d(M_t^f/X_{t-1}^*) &= d(\tilde{M}_t^f(X_{t-1}/X_{t-1}^*)) \\ &= d\tilde{M}_t^f + \tilde{M}_{ss}^f\, d(X_{t-1}/X_{t-1}^*) \\ &= d\tilde{M}_t^f + \tilde{M}_{ss}^f\, d\left(\frac{g_0}{g^*}\frac{g_1}{g^*}\cdots\frac{g_{t-1}}{g^*}\right) \\ &= d\tilde{M}_t^f + \frac{\tilde{M}_{ss}^f}{g^*}\left(\sum_{j=0}^{t-1} dg_j\right). \end{aligned}$$

By dividing $d(M_t^f/X_{t-1}^*)$ with $\tilde{M}_{ss}^f$, I obtain $IRF_{M^f}^{ratio}(t)$.

D.3 Stock Variables

Stock variables in the original equilibrium, such as K_t, A_t, B_t, D_t, and F_t, exhibit a stochastic trend and are detrended by X_t in the detrended equilibrium. For the purpose of this paper, I must also compute the impulse responses of the stock variables in terms of their *ratio* deviation from the balanced growth path.

The impulse responses of the stock variables can be computed in a similar way as those of the flow variables. Let M_t^s denote one of the stock variables in the original equilibrium, $\tilde{M}_t^s := M_t^s/X_t$ denote its detrended variable, and $\tilde{M}_{ss}^s$ denote the steady state value of $\tilde{M}_t^s$ in the detrended equilibrium. M_t^s on the balanced growth path, which I denote as M_t^{s*}, is determined by

$$M_t^{s*} = \tilde{M}_{ss}^s X_t^*, \quad t \geq 0.$$

(See Online Appendix D.2 for the definition of X_t^*.) The impulse response of M_t^s in terms of their ratio deviation from the balanced growth path can now be written as

$$IRF_{M^s}^{ratio}(t) = \frac{M_t^s - M_t^{s*}}{M_t^{s*}} = \frac{M_t^s/X_t^* - \tilde{M}_{ss}^s}{\tilde{M}_{ss}^s}.$$

After obtaining $d\tilde{M}_t^s = \tilde{M}_t^s - \tilde{M}_{ss}^s$ by solving the detrended equilibrium with Auclert et al. (2021)'s method, I compute $d(M_t^s/X_t^*) = M_t^s/X_t^* - \tilde{M}_{ss}^s$ using the following first-order-approximated equation.

$$d(M_t^s/X_t^*) = d\tilde{M}_t^s + \frac{\tilde{M}_{ss}^s}{g^*}\left(\sum_{j=0}^{t} dg_j\right).$$

By dividing $d(M_t^s/X_t^*)$ with $\tilde{M}_{ss}^s$, I obtain $IRF_{M^s}^{ratio}(t)$.

E Truncation Errors

Auclert et al. (2021)'s sequence space approach requires a truncation of sequences, and truncation errors can be nontrivial when the economy is extremely persistent. In this section, I inspect truncation errors and verify that at the posterior mode, the truncation errors are negligible in the model statistics used in this paper.

In this paper, I truncate sequences at $T = 700$ when solving models and drop the last seven periods further when evaluating moments. I start by comparing the impulse response sequences (at the posterior mode) obtained under truncation at $T = 700$ with those under truncation at $T = 1500$. Three observations emerge from this comparison: i) in each model (*i.e.*, each of RASOE (z,g,μ), HASOE (z,g,μ), and HASOE (z,g,μ,η) models), all impulse responses reach a balanced growth path before $t = 1499$; ii) there are some impulse responses that have not reached a balanced growth path in $t = 699$; iii) even in such cases, the impulse response sequences obtained under truncation at $T = 700$ are distorted only in the last few periods compared to those under truncation at $T = 1500$.

As an example, in Figure E.1, I plot the impulse responses of Y_t, C_t, I_t, and TB_t/Y_t to a trend shock in the RASOE model evaluated at its posterior mode, where the trend shock is very persistent ($\rho_g = 0.988$). All the impulse responses in this figure reach a new balanced growth path before $t =$ 1499 (observation i)). The impulse responses of Y and C have already reached a balanced growth

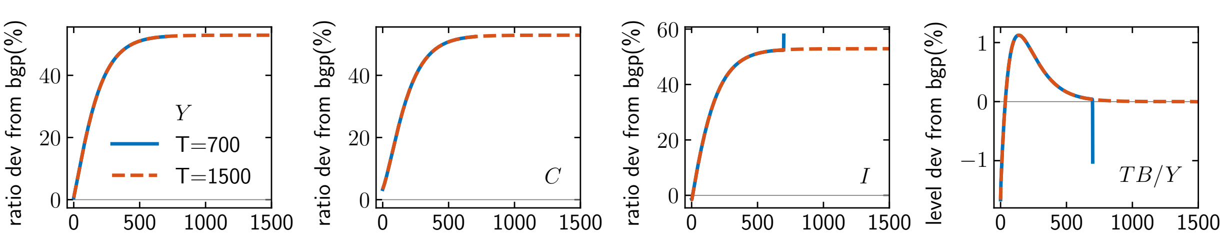


Figure E.1: Truncation Errors on the Impulse Responses to g Shock in the RASOE (z,g,μ) Model

Notes: This figure plots the impulse responses of Y, C, I, and TB/Y to a one-standard-deviation g shock in the RASOE (z,g,μ) model evaluated at the posterior mode. In particular, the model is solved under two different truncation lengths, $T = 700$ and $T = 1500$.

path before $t = 699$, while those of I and TB/Y have not (observation ii)). By truncating sequences at $T = 700$ in which I and TB/Y have not reached a balanced growth path yet, a truncation error occurs in their impulse responses. Importantly, however, the truncation error occurs only in the last few periods (observation iii)).

Based on these observations, when I evaluate model moments, I drop the last $0.01T$ periods from the length-T sequences and use only the first $0.99T$ periods. In the rest of the section, I evaluate the model statistics used in this paper at the posterior mode of each model under two different truncation lengths, $T = 700$ and 1500, and compare them. Specifically, I compare the following model statistics: i) Figures E.2, E.3, and E.4 compare the autocovarinaces of observable time series $[\Delta \log Y_t, \Delta \log C_t, \Delta \log I_t, \Delta TB_t/Y_t]$ (which are used when evaluating a likelihood in the Bayesian estimation); ii) Table E.1 compares business cycle moments; and iii) Table E.2 compares the variance decomposition result. From these comparisons, I find that truncation errors on these model statistics are negligible.

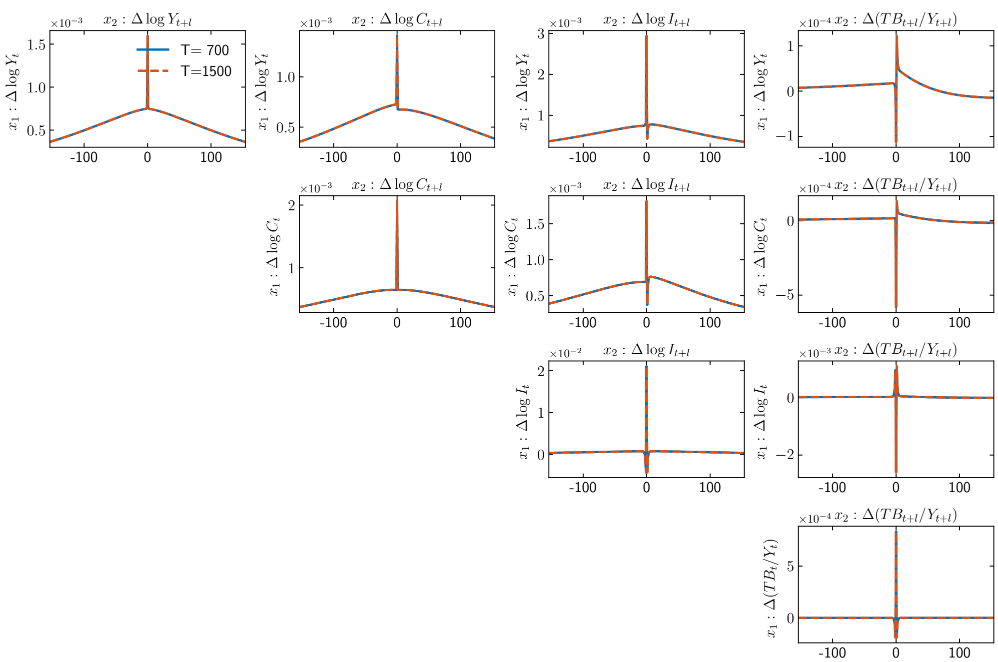


Figure E.2: Truncation Errors on the Autocovariances of Observables: RASOE (z, g, μ) Model

Notes: The RASOE (z, g, μ) model is evaluated at the posterior mode and solved at two different truncation lengths, $T = 700$ and 1500. The moments are evaluated using the first $0.99T$ periods of the length-T sequences.

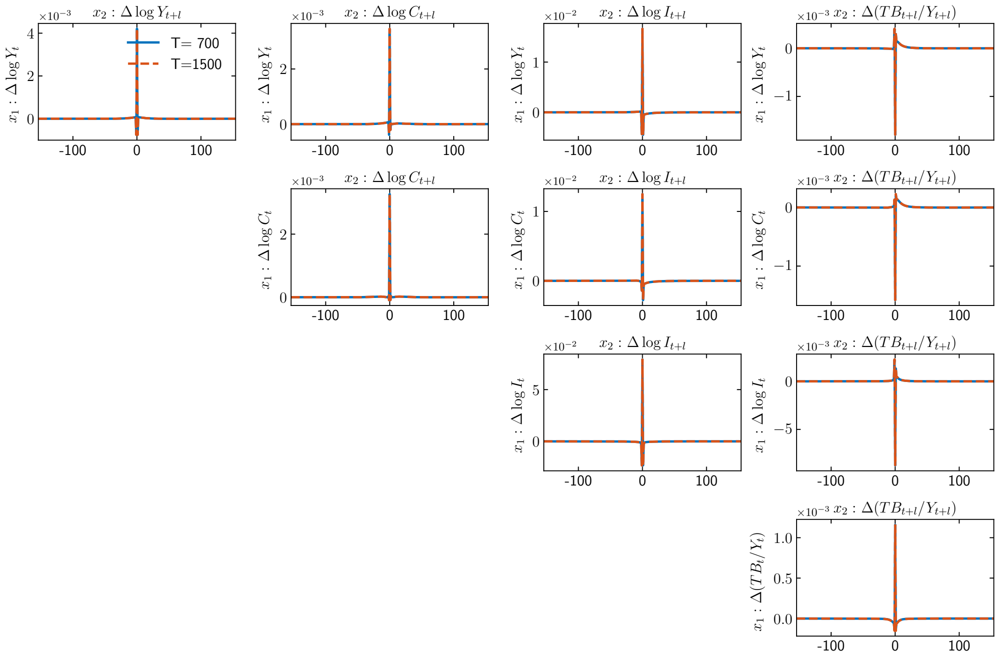


Figure E.3: Truncation Errors on the Autocovariances of Observables: HASOE (z, g, μ) Model

Notes: The HASOE (z, g, μ) model is evaluated at the posterior mode and solved at two different truncation lengths, $T = 700$ and 1500. The moments are evaluated using the first $0.99T$ periods of the length-T sequences.

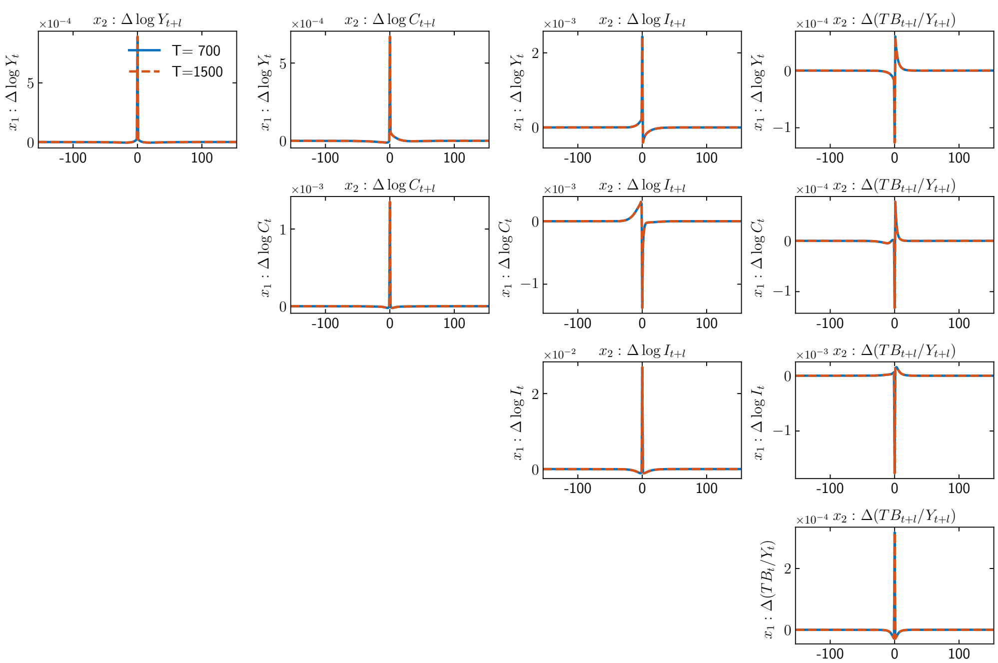


Figure E.4: Truncation Errors on the Autcovariances of Observables: HASOE (z, g, μ, η) Model

Notes: The HASOE (z, g, μ, η) model is evaluated at the posterior mode and solved at two different truncation lengths, $T = 700$ and 1500. The moments are evaluated using the first $0.99T$ periods of the length-T sequences.

Table E.1: Truncation errors on business cycle moments

			$\Delta \log Y_t$	$\Delta \log C_t$	$\Delta \log I_t$	$\Delta(TB_t/Y_t)$
Standard deviation						
	RASOE (z,g,μ) model	$T = 700$	0.040	0.045	0.145	0.029
		$T = 1500$	0.040	0.045	0.145	0.029
	HASOE (z,g,μ) model	$T = 700$	0.065	0.057	0.281	0.034
		$T = 1500$	0.065	0.057	0.281	0.034
	HASOE (z,g,μ,η) model	$T = 700$	0.030	0.037	0.164	0.018
		$T = 1500$	0.030	0.037	0.164	0.018
Contemporaneous correlation						
with $\Delta \log Y_t$	RASOE (z,g,μ) model	$T = 700$		0.776	0.507	-0.097
		$T = 1500$		0.776	0.507	-0.097
	HASOE (z,g,μ) model	$T = 700$		0.932	0.911	-0.814
		$T = 1500$		0.932	0.912	-0.815
	HASOE (z,g,μ,η) model	$T = 700$		0.611	0.497	-0.237
		$T = 1500$		0.611	0.497	-0.237
with $\Delta(TB_t/Y_t)$	RASOE (z,g,μ) model	$T = 700$		-0.443	-0.620	
		$T = 1500$		-0.443	-0.620	
	HASOE (z,g,μ) model	$T = 700$		-0.811	-0.913	
		$T = 1500$		-0.812	-0.913	
	HASOE (z,g,μ,η) model	$T = 700$		-0.202	-0.603	
		$T = 1500$		-0.202	-0.603	
with $\Delta \log C_t$	RASOE (z,g,μ) model	$T = 700$			0.275	
		$T = 1500$			0.275	
	HASOE (z,g,μ) model	$T = 700$			0.776	
		$T = 1500$			0.776	
	HASOE (z,g,μ,η) model	$T = 700$			-0.228	
		$T = 1500$			-0.228	
Autocorrelation						
with lag 1	RASOE (z,g,μ) model	$T = 700$	0.472	0.314	-0.204	-0.228
		$T = 1500$	0.472	0.314	-0.204	-0.228
	HASOE (z,g,μ) model	$T = 700$	-0.180	-0.029	-0.292	-0.126
		$T = 1500$	-0.180	-0.030	-0.295	-0.132
	HASOE (z,g,μ,η) model	$T = 700$	0.026	-0.002	-0.027	-0.091
		$T = 1500$	0.026	-0.002	-0.027	-0.091
with lag 2	RASOE (z,g,μ) model	$T = 700$	0.469	0.314	-0.051	-0.082
		$T = 1500$	0.469	0.315	-0.051	-0.082
	HASOE (z,g,μ) model	$T = 700$	0.008	0.009	-0.029	-0.050
		$T = 1500$	0.008	0.008	-0.030	-0.054
	HASOE (z,g,μ,η) model	$T = 700$	0.020	-0.013	-0.040	-0.079
		$T = 1500$	0.020	-0.013	-0.040	-0.079
with lag 3	RASOE (z,g,μ) model	$T = 700$	0.468	0.315	0.004	-0.029
		$T = 1500$	0.468	0.315	0.004	-0.029
	HASOE (z,g,μ) model	$T = 700$	0.015	0.001	-0.014	-0.045
		$T = 1500$	0.015	0.001	-0.016	-0.049
	HASOE (z,g,μ,η) model	$T = 700$	0.014	-0.016	-0.042	-0.068
		$T = 1500$	0.014	-0.016	-0.042	-0.068

Notes: Each model is evaluated at its posterior mode and solved at two different truncation lengths, $T = 700$ and 1500. The moments are evaluated using the first $0.99T$ periods of the length-T sequences.

Table E.2: Truncation errors on variance decomposition

		$\Delta \log Y_t$	$\Delta \log C_t$	$\Delta \log I_t$	$\Delta(TB_t/Y_t)$
RASOE (z, g, μ) *model*					
stationary productivity shock (z)	$T = 700$	0.498	0.164	0.319	0.013
	$T = 1500$	0.498	0.164	0.319	0.013
trend shock (g)	$T = 700$	0.502	0.835	0.053	0.344
	$T = 1500$	0.502	0.835	0.053	0.344
interest rate shock (μ)	$T = 700$	0.001	0.000	0.628	0.642
	$T = 1500$	0.001	0.000	0.628	0.642
HASOE (z, g, μ) *model*					
stationary productivity shock (z)	$T = 700$	0.526	0.824	0.215	0.352
	$T = 1500$	0.526	0.824	0.213	0.348
trend shock (g)	$T = 700$	0.473	0.166	0.729	0.385
	$T = 1500$	0.473	0.166	0.730	0.387
interest rate shock (μ)	$T = 700$	0.000	0.010	0.057	0.264
	$T = 1500$	0.000	0.010	0.057	0.265
HASOE (z, g, μ, η) *model*					
stationary productivity shock (z)	$T = 700$	0.924	0.325	0.173	0.008
	$T = 1500$	0.924	0.325	0.173	0.008
trend shock (g)	$T = 700$	0.069	0.032	0.352	0.946
	$T = 1500$	0.069	0.032	0.352	0.946
interest rate shock (μ)	$T = 700$	0.000	0.000	0.009	0.036
	$T = 1500$	0.000	0.000	0.009	0.036
financial friction shock (η)	$T = 700$	0.007	0.643	0.466	0.009
	$T = 1500$	0.007	0.643	0.466	0.009

Notes: Each model is evaluated at its posterior mode and solved at two different truncation lengths, $T = 700$ and 1500. The moments are evaluated using the first $0.99T$ periods of the length-T sequences.

F MPC Estimation using Micro Data

F.1 Method

Following Hong (2022), I estimate MPC out of transitory income shocks using an extended version of Blundell et al. (2008). Let the individual earnings $Y_{i,t}$ be specified as follows.

$$
\begin{aligned}
\log Y_{i,t} &= Z_{i,t}'\varphi_t + P_{i,t} + \varepsilon_{i,t}, \\
P_{i,t} &= \rho P_{i,t-1} + \zeta_{i,t}, \\
\zeta_{i,t} \sim_{iid} (0, \sigma_{ps}^2), \quad & \varepsilon_{i,t} \sim_{iid} (0, \sigma_{tr}^2), \quad \text{and} \quad (\zeta_{i,t})_t \perp (\varepsilon_{i,t})_t,
\end{aligned}
$$

where $(x_t)_t$ represents time series $(\cdots, x_{t-1}, x_t, x_{t+1}, \cdots)$. $Z_{i,t}'\varphi_t$ represents the predictable component of log earnings $\log Y_{i,t}$, where $Z_{i,t}$ denotes a vector of dummy variables for observable characteristics of household i.[5] The unpredictable components of log earnings, $y_{i,t} (:= \log Y_{i,t} - Z_{i,t}'\varphi_t)$, is composed of a persistent component $P_{i,t}$, which follows an AR(1) process, and a transitory component, which is an *i.i.d.* shock. This earnings process specification is consistent with the model in subsection 2.2 such that $Z_{i,t}'\varphi_t$, $P_{i,t}$, and $\varepsilon_{i,t}$ correspond to $\log(w_t \Gamma_i \bar{l}_t)$, $e_{1,i,t}$, and $e_{2,i,t}$, respectively.

Let $c_{i,t}$ be the unpredictable component of consumption (*i.e.*, $c_{i,t} := \log C_{i,t} - Z_{i,t}'\varphi_t^c$, where $C_{i,t}$ is consumption and $Z_{i,t}'\varphi_t^c$ is the predictable component of $\log C_{i,t}$). Blundell et al. (2008)'s partial insurance parameter to transitory shocks for a group G, which I denote by ψ_G, is defined as follows.

$$
\psi_G = \frac{cov[\Delta c_{i,t}, \varepsilon_{i,t} | (i,t) \in G]}{cov[\Delta y_{i,t}, \varepsilon_{i,t} | (i,t) \in G]}.
$$

In other words, ψ_G is the elasticity of consumption with respect to earnings when the earnings change is caused by an idiosyncratic transitory shock.

As in Online Appendix G.6 of Hong (2022), I estimate ψ_G adopting Kaplan and Violante (2010)'s identification strategy for Blundell et al. (2008)'s partial insurance parameters under the 'AR(1)+*i.i.d.*' specification of the earnings process. Specifically, let $\tilde{\Delta}^K y_{i,t}$ and $\Delta^K c_{i,t}$ be

$$
\begin{aligned}
\tilde{\Delta}^K y_{i,t} &:= y_{i,t} - \rho^K y_{i,t-K}, \quad K \geq 1, \quad \text{and} \\
\Delta^K c_{i,t} &:= c_{i,t} - c_{i,t-K}, \quad K \geq 1.
\end{aligned}
$$

[5] The observable characteristics of households include education, ethnicity, employment status, region, cohort, household size, number of children, urban area, the existence of members other than heads and spouses earning income, and the existence of persons who do not live with but are financially supported by the household. Among these characteristics, education, ethnicity, employment status, and region are allowed to have time-varying effects.

Then, we have

$$\tilde{\Delta}^K y_{i,t} = \sum_{s=0}^{K-1} \rho^s \zeta_{i,t-s} + \varepsilon_{i,t} - \rho^K \varepsilon_{i,t-K}.$$

When the grouping of observations is independent of $(\zeta_{i,t+j}, \varepsilon_{i,t+j})_{j\geq 0}$ and $\Delta c_{i,t}$ is independent of $(\zeta_{i,t+j}, \varepsilon_{i,t+j})_{j\geq 1}$, we can derive

$$\psi_G = \frac{cov[\Delta^K c_{i,t}, \tilde{\Delta}^K y_{i,t+K} | (i,t) \in G]}{cov[\tilde{\Delta}^K y_{i,t}, \tilde{\Delta}^K y_{i,t+K} | (i,t) \in G]}. \tag{F.1}$$

To identify ψ_G using equation (F.1), we need the value of ρ. Adopting Floden and Lindé (2001)'s identification strategy, parameter ρ is estimated using the following moment conditions for the autocovariances of $y_{i,t}$.[6]

$$E[y_{i,t}^2] = \frac{\sigma_{ps}^2}{1-\rho^2} + \sigma_{tr}^2,$$
$$E[y_{i,t}, y_{i,t+nK}] = \frac{\sigma_{ps}^2}{1-\rho^2}\rho^{nK}, \quad n \geq 1.$$

Once ρ is estimated, I estimate ψ_G using equation (F.1). Since ψ_G is an elasticity, I transform it to MPC by multiplying a group-level consumption-income ratio as follows.

$$MPC_G = \psi_G \frac{E[C_{i,t} | (i,t) \in G]}{E[Y_{i,t} | (i,t) \in G]}. \tag{F.2}$$

ENAHO provides the year-over-year growth of quarterly income and consumption, and thus, I set one period as a quarter and $K = 4$ for the Peruvian sample. As a result, I obtain quarterly MPCs of Peruvian households. On the other hand, the PSID provides the two-year-over-two-year growth of annual income and consumption, and thus, I set one period as a year and $K = 2$ for the U.S. sample. As a result, I obtain annual MPCs of U.S. households.

F.2 Revisions on Hong (2022)

Compared to Hong (2022), I make three revisions to the MPC estimation procedure, which are necessary to maintain consistency between micro moments and macro data or model. First, I change the consumption measure from non-durable consumption to total consumption (including both non-durable and durable consumption). Once the model is calibrated by targeting the MPC moments, I estimate the model using macro data. In this step, I use total consumption series (as studies on emerging market business cycles typically do) because non-durable consumption is not

[6] In this estimation, I obtain the estimates of ρ, σ_{ps}, and σ_{tr}. These estimates for Peruvian households are also used in subsection 3.1 to calibrate the earnings process in the HASOE model.

available in the Peruvian national accounts. To make the consumption concept consistent between micro and macro data, I use the total consumption measure when analyzing the micro data, too.

Second, the sample periods are changed for both ENAHO and the PSID because some of the key durable expenses are available only after certain years in both surveys. Specifically, I use the 2011-2018 waves of ENAHO and the 2005-2017 waves of the PSID.[7]

Third, the earnings process specification is revised to be consistent with the HASOE model. Blundell et al. (2008)'s method requires a structural specification of the earnings process. In its baseline specification, Hong (2022) assumes that the unpredictable component of earnings is composed of a permanent component and a transitory component, where the permanent component follows a random walk as in the original specification of Blundell et al. (2008). In this paper, I replace the random walk component with a component following an AR(1) process so that the earnings process specification imposed in the MPC estimation is consistent with the model.[8]

F.3 Variable Construction

The consumption measure in MPC estimation is total consumption, which includes both non-durable and durable consumption. I construct such consumption by aggregating the following expenses in each of ENAHO and the PSID: non-durable expenses including 1) food, 2) clothing (including clothing services, footwear, watches and jewelry), 3) housing rent, rental equivalence of owned or donated housing, 4) utilities (heat, electricity, water, etc.), 5) telephone and cable, 6) vehicle repairs and maintenance, 7) gasoline and oil, 8) parking, 9) public transportation, 10) household repairs and maintenance, 11) recreation, 12) insurance (home insurance, car insurance, health insurance, etc.), 13) childcare, 14) domestic services and other home services, 15) personal care, 16) alcohol, 17) tobacco, and 18) daily non-durables (laundry items, bathroom items, matches, candle, stationeries, etc.), and durable expenses including 19) vehicles, 20) furnishings and equipment (textiles, furniture, floor coverings, appliances, housewares, etc.), 21) health, and

[7]My ENAHO sample starts from 2011 for the following reason. ENAHO is conducted continuously (*i.e.*, households are interviewed in different months) and the reference periods of income and expense items are usually in the format of a 'specified period before the interview' (such as 'previous *n* months') rather than a fixed calender period (such as 'during 2014'). Accordingly, I set the reference periods of my consumption and income measures using the same format (*i.e.*, a specified period before the interview such as 'previous *n* months'). One exception is Questionnaire 612. This questionnaire collects information on household furnishings, equipment, and vehicles, which take a sizable portion of durable goods. Until 2010, this questionnaire asks which calendar year each item is acquired, and thus it is impossible to aggregate this questionnaire's expense items with other expense items under a consistent reference period format. From 2011 onward, Questionnaire 612 asks the acquisition month instead of the acquisition year, which makes it possible to recover this questionnaire's expense items during a specified period before the interview (such as 'previous *n* months') and to aggregate these expense items with other expense items under a consistent reference period format. My PSID sample starts from 2005 because the survey began to collect expenses on household furnishings and equipment since then. Moreover, some non-durable items including clothing and recreation are also collected from 2005 onward.

[8]Hong (2022) also considers the 'AR(1) + *i.i.d.*' specification during a robustness check in his Appendix G.6.

22) education.[9] Among the listed expenses, ENAHO does not have expenses on 13) childcare, and the PSID does not have expenses on 14) domestic services and other home services, 15) personal care, 16) alcohol, 17) tobacco, and 18) daily non-durables (laundry items, bathroom items, matches, candle, stationeries, etc.). Nonpurchased consumption, such as donations, food stamps, in-kind income, and self-production, is excluded.

The income measure in MPC estimation is the sum of disposable labor income and transfers, as in Blundell et al. (2008). Capital income is excluded in order not to falsely attribute endogenous capital income changes as income shocks. In ENAHO, capital income and labor income are not distinguishable in self-employment income. As in Diaz-Gimenez et al. (1997), Krueger and Perri (2006), and Hong (2022), I split the self-employment income into labor income and capital income parts using the ratio between unambiguous capital and labor incomes in the sample.[10] In ENAHO, imputed components of missing income are distinguishable, and I exclude them from Peruvian incomes, as in Hong (2022). For the PSID sample, I closely follow Kaplan et al. (2014) in constructing U.S. incomes. Specifically, U.S. households' disposable labor income and transfers are constructed by i) estimating federal income taxes for total income (including labor income, transfers, and capital income) by TAXSIM program, ii) splitting proportionately the estimated federal taxes into the labor income and transfers part and the capital income part, and iii) subtracting the federal taxes on labor income and transfers from gross labor income and transfers.

In ENAHO, reference periods vary across income and expense items. Importantly, Peruvian households report 97.5% of income items and 92.9% of expense items (in value) under reference periods shorter than or equal to the previous three months, on average. Given this feature of the data, I set the reference period of Peruvian income and consumption as the previous three months. Expense and income items reported under a different reference period than the previous three months are scaled to three-month expenses and incomes, respectively. (For example, a monthly tobacco expense is scaled up by a factor of three.)[11] Moreover, to remove any comovement between income and consumption that occurs prior to the previous three months, I exclude income items with reference periods longer than the previous three months from Peruvian incomes.

In the PSID, the reference periods of income items are firmly fixed to a calendar year, while the reference periods of expense items can depend on interpretation, as Crawley (2020) notes. For example, food expenses in the PSID can be interpreted either as the last week's expense or

[9] In listing the expenses, I categorize expenses on 21) health and 22) education as durable expenses because of their durable nature. In national accounts, however, they are categorized as non-durable consumption. Since I use total consumption, how they are categorized between durable and non-durable consumption does not have any effect.

[10] As noted in footnote 24, the ratio of (unambiguous labor income) / (unambiguous labor income + unambiguous capital income) is 0.817 in ENAHO, and this ratio is close to the ratio that Diaz-Gimenez et al. (1997) and Krueger and Perri (2006) use for their U.S. sample, 0.864.

[11] Online Appendix B.2 of Hong (2022) describes how one can achieve such scaling effectively using certain variables in the ENAHO data.

the average weekly expense during the calendar year. I adopt the latter interpretation, as related studies often do, and treat the reference periods of expense items as being synchronized with those of income items. Accordingly, I set the reference period of U.S. income and consumption as the corresponding calender year.

ENAHO is conducted annually, and I use the 2011-2018 waves. This ENAHO sample provides seven years of the year-over-year growth of quarterly income and consumption. For the PSID, I use the 2005-2017 waves, and the survey is conducted biannually during the sample period. This PSID sample provides six years of the two-year-over-two-year growth of annual income and consumption.

In both the ENAHO and the PSID samples, nominal income and consumption are deflated with the Consumer Price Index (CPI) series.[12]

F.4 Sample Selection

The sample selection for ENAHO closely follows Hong (2022) and proceeds as follows. First, I drop observations when households appear only once in the survey. Second, I drop observations when households are interviewed in different months between two consecutive surveys or when household heads are changed. I also drop observations when it is likely that two different households are linked as panel observations by failing to distinguish an old household moving out and a new household moving into the same address.[13] Third, I drop observations classified as 'incomplete' by pollsters. Fourth, I drop observations when household heads are younger than 25 and older than 65. Fifth, I drop observations when observable characteristics used to control for the predictable components of income and consumption are missing. Sixth, I drop observations reporting nonpositive income or consumption. Seventh, I drop observations with too much imputed value or too much value reported under a longer reference period than the previous three months in their income.[14] Eighth, I drop income outliers.[15] As a result, I obtain a sample composed of 36,292 observations, 18,479 pairs of two consecutive observations, and 7,241 triplets of three consecutive observations. The panel A of Table F.1 reports the number of remaining observations in each step.

[12]Unlike the reference periods in the PSID sample, the reference periods in the ENAHO sample are not fixed to a calendar period. For example, the three-month reference period of households surveyed in January, 2015 starts one-month earlier than that of households surveyed in February, 2015. Fortunately, this feature of the data does not complicate the deflation procedure, as ENAHO provides variables recording within-year-deflated values of income and expense items. Online Appendix B.2 of Hong (2022) provides a detailed deflation procedure using these variables.

[13]Hong (2022) defines such panel observations as 'potentially fake panel observations.' The potentially fake panel observations can be effectively detected and dropped by using household-member-level information. See Online Appendix B.4 of Hong (2022) for a detailed discussion.

[14]'Too much value' is defined as follows. For each $(x, y) \in \{$(imputed value, baseline income) (value reported under a reference period longer than the previous three months, baseline income)$\}$, I drop observations when $\frac{x}{x+y} > 0.05$.

[15]Income outliers are defined as households exhibiting an extreme income growth, which falls in the range of extreme 1 % (0.5% at the top and 0.5% at the bottom) in a calendar-year subsample at least once.

Table F.1: Sample Selection

	N_1	N_2	N_3
A. ENAHO			
households appearing only once	87,305	59,691	32,077
months not matched, fake panel obs., or head changed	73,248	47,950	22,652
incomplete survey	63,410	38,343	17,386
age restriction, 25-65	48,636	28,983	12,971
observable characteristics missing	48,403	28,904	12,955
nonpositive Y and C	48,052	28,522	12,732
too much imputation or 3ml in Y	36,805	18,808	7,398
income outliers	36,292	18,479	7,241
B. The PSID			
households appearing only once or head changed	57,560	45,553	33,546
SEO sample 1968 and Latino sample 1990/1992	39,660	31,523	23,386
topcoded obs.	39,650	31,507	23,369
age restriction, 25-65	31,447	24,380	17,711
observable characteristics missing	30,225	23,277	16,805
non-positive Y and C	30,028	23,021	16,570
income outliers	29,145	22,345	16,092

Notes: In the penultimate line of panel A, '3ml' is an abbreviation for 'items with reference periods longer than the previous three months.' The columns labeled N_1, N_2, and N_3 report the number of remaining observations, pairs of two consecutive observations, and triplets of three consecutive observations, respectively, in each step.

The sample selection for the PSID proceeds similarly to the sample selection for ENAHO as follows. First, I drop observations when households appear only once in the survey. I also drop observations when household heads are changed. Second, I drop observations if they belong to the sample from Survey of Economic Opportunities (SEO) (added to the PSID in 1968) or to the Latino sample (added to the PSID in 1990 and 1992). Third, I drop observations when their income or consumption include topcoded values. Fourth, I drop observations when household heads are younger than 25 and older than 65. Fifth, I drop observations when observable characteristics used to control for the predictable components of income and consumption are missing. Sixth, I drop observations reporting nonpositive income or consumption. Seventh, I drop income outliers, which are defined in the same way as in the Peruvian sample selection. As a result, I obtain a sample composed of 29,145 observations, 22,345 pairs of two consecutive observations, and 16,092 triplets of three consecutive observations. The panel B of Table F.1 reports the number of remaining observations in each step.

F.5 Earnings Grouping

When disciplining the HASOE model, I use MPC estimated within each decile of residual earnings, which is $e_{i,t}$ in the model and $y_{i,t}$ in the MPC estimation procedure described in Online Appendix F.1. In particular, I do not group observations by total earnings, which is $w_t \Gamma_i e_{i,t} \bar{l}_t$ in the model and $Y_{i,t}$ in the MPC estimation procedure in Online Appendix F.1, because $e_{i,t}$ bears risk and thus induces precautionary saving and MPC heterogeneity, while Γ_i does not.

When estimating the MPC moments, I construct residual earnings deciles in the same way as in Hong (2022). See section 3.4 of the paper for details.

G The Market Value of Wealth

In subsection 3.1, I calibrate parameter p such that the entrepreneurs' wealth share matches top $100(1-p)\%$ share of wealth in data. For this purpose, I use wealth inequality data from World Inequality Database (WID). As Alvaredo et al. (2021) note, WID's wealth concept is a market-value of wealth, which includes financial assets yielding nonproductive pure rents, such as entrepreneurs' claims to rents R_t^E in the model. Thus, I evaluate the market value of the claims assuming that entrepreneurs can trade the claims among themselves and include this value as part of entrepreneurs' wealth. Specifically, the market value of the claims is evaluated as follows.

$$
\begin{aligned}
U_t^E &:= \sum_{s=1}^{\infty} (\beta^E)^s \frac{\lambda_{t+s}^E}{\lambda_t^E} R_{t+s}^E \\
&= \sum_{s=1}^{\infty} \frac{Q_{0,t+s}}{Q_{0,t}} \frac{\chi_{t+s}^{agg} + \xi(1+r_{t+s}^b) B_{t+s-1}}{1-p}.
\end{aligned}
$$

Let $\tilde{U}_t^E := U_t^E / X_t$. Then, we have

$$
\tilde{U}_t^E = \sum_{s=1}^{\infty} \frac{Q_{0,t+s}}{Q_{0,t}} \frac{\tilde{\chi}_{t+s}^{agg} + \xi(1+r_{t+s}^b) \tilde{B}_{t+s-1}}{1-p} \frac{X_{t+s-1}}{X_t}.
$$

In the steady state of the detrended equilibrium, $\tilde{U}_t^E$ becomes

$$
\tilde{U}_{ss}^E = \sum_{s=1}^{\infty} \frac{1}{(1+r_{ss})^s} \frac{\tilde{\chi}_{ss}^{agg} + \xi(1+r_{ss}) \tilde{B}_{ss}}{1-p} (g_{ss})^{s-1}
$$

$$
\Leftrightarrow \tilde{U}_{ss}^E = \frac{\tilde{\chi}_{ss}^{agg} + \xi(1+r_{ss}) \tilde{B}_{ss}}{1+r_{ss}-g_{ss}} \frac{1}{1-p}. \tag{G.1}
$$

In the detrended steady state, the market value of entrepreneurs' wealth, $\tilde{\mathscr{W}}_{ss}^E$, and that of workers' wealth, $\tilde{\mathscr{W}}_{ss}^{W}$, are determined as follows.

$$\tilde{\mathscr{W}}_{ss}^{E} = \tilde{U}_{ss}^{E} + \tilde{A}_{ss}^{E}, \quad \text{and} \tag{G.2}$$

$$\tilde{\mathscr{W}}_{ss}^{W} = \tilde{B}_{ss}^{W} + \tilde{A}_{ss}^{W}. \tag{G.3}$$

The market value of the total wealth in this economy, $\tilde{\mathscr{W}}_{ss}$, is characterized as follows.

$$\begin{aligned}\tilde{\mathscr{W}}_{ss} = & \underbrace{p\tilde{\mathscr{W}}_{ss}^{W}}_{\text{workers' portion}} + \underbrace{(1-p)\tilde{\mathscr{W}}_{ss}^{E}}_{\text{entreprenerus' portion}} \\ = & \tilde{A}_{ss} + \tilde{B}_{ss} + (1-p)\tilde{U}_{ss}^{E} \\ = & \tilde{K}_{ss} - \tilde{D}_{ss} + (1-p)\tilde{U}_{ss}^{E} \quad \text{(by equation (B.64))}.\end{aligned}$$

$$\tilde{\mathscr{W}}_{ss} = \tilde{K}_{ss} - \tilde{D}_{ss} + \frac{\tilde{\chi}_{ss}^{agg} + \xi(1+r_{ss})\tilde{B}_{ss}}{1 + r_{ss} - g_{ss}}. \tag{G.4}$$

Parameter p is calibrated such that the fraction $(1-p)\tilde{\mathscr{W}}_{ss}^{E}/\tilde{\mathscr{W}}_{ss}$ in the model under a given value of p is equal to the top $100(1-p)\%$ share of wealth in the data (WID).

H Consumption Response Decomposition in the RASOE Model

In Figure 2a, I decompose the response of GHH_t to a trend shock (g) in the RASOE model into the response driven by w_t and $h_t(\cdot)$ and the response driven by r_t^a. This consumption response decomposition, however, cannot be obtained directly from the sequence space approach. This is because I must write the DAG representation of the RASOE model (presented in Figure C.2) such that households' partial equilibrium problem does not form a separate block for the following reason: if the DAG representation of the RASOE model were written such that representative households' partial equilibrium problem form a separate block, the Jacobian of the block contains $(\partial GHH_s/r_t^a)_{s,t\geq 0}$, which, for any t, never dies out when $s \to \infty$.[16]

Thus, I obtain the consumption response decomposition in the RASOE model using the conventional state space approach developed by Blanchard and Kahn (1980) as follows. Let

$$dy_t = d\tilde{C}_t, \quad dx_t = [d\tilde{A}_{t-1}, dg_t, d\tilde{w}_t, dr_t^a]', \quad \text{and} \quad d\varepsilon_t = [dg_t, d\tilde{w}_t, dr_t^a]'.$$

The households' partial equilibrium is characterized by detrended equilibrium conditions (A.12),

[16] Workers' partial equilibrium problem in the HASOE model does not have this issue, and thus, the DAG representation of the HASOE model (presented in Figure C.1) is written such that workers' problem forms a separate block. As a result, workers' consumption response decomposition across driving factors are obtained directly from the sequence space approach in the HASOE model.

(A.13), (A.14), and (A.15). After substituting out L_t and $\tilde{\lambda}_t$ from the equilibrium conditions, the households' partial equilibrium can be described by the following VAR representation.

$$A \cdot \begin{bmatrix} dy_{t+1} \\ dx_{t+1} \end{bmatrix} = B \cdot \begin{bmatrix} dy_t \\ dx_t \end{bmatrix} + C \cdot d\varepsilon_{t+1},$$

where A and B are 5-by-5 matrices and C is a 5-by-3 matrix. Note that dy_t is a control variable, dx_t is a vector of state variables, and $d\varepsilon_t$ is a vector of exogenous variables in this system. A is invertible under any posterior parameter draws, and thus, this equation can be rewritten as follows.

$$\begin{bmatrix} dy_{t+1} \\ dx_{t+1} \end{bmatrix} = M \cdot \begin{bmatrix} dy_t \\ dx_t \end{bmatrix} + A^{-1}C \cdot d\varepsilon_{t+1},$$

where $M := A^{-1}B$. Let $M = W^{-1}JW$ be the Jordan decomposition of M under which the diagonal vector of J, $(\lambda_1, \cdots, \lambda_5)$, satisfies $|\lambda_1| \geq \cdots \geq |\lambda_5|$. I find that there is only one explosive eigenvalue (*i.e.*, $|\lambda_1| > 1 \geq |\lambda_2| \geq \cdots \geq |\lambda_5|$) under any posterior parameter draws, and thus, Blanchard and Kahn (1980)'s stationarity condition is always satisfied. For notational convenience, I introduce submatrices of J, W, and $WA^{-1}C$ as follows.

$$J = \begin{bmatrix} \underbrace{J_1}_{1\times1} & \underbrace{0}_{1\times4} \\ \underbrace{0}_{4\times1} & \underbrace{J_2}_{4\times4} \end{bmatrix}, \quad W = \begin{bmatrix} \underbrace{W_{11}}_{1\times1} & \underbrace{W_{12}}_{1\times4} \\ \underbrace{W_{21}}_{4\times1} & \underbrace{W_{22}}_{4\times4} \end{bmatrix}, \quad \text{and} \quad WA^{-1}C = \begin{bmatrix} \underbrace{T_1}_{1\times3} \\ \underbrace{T_2}_{4\times1} \end{bmatrix}.$$

Let

$$\begin{bmatrix} dz_{1,t} \\ dz_{2,t} \end{bmatrix} := W \begin{bmatrix} dy_t \\ dx_t \end{bmatrix} = \begin{bmatrix} W_{11} \cdot dy_t + W_{12} \cdot dx_t \\ W_{21} \cdot dy_t + W_{22} \cdot dx_t \end{bmatrix}. \tag{H.1}$$

Then, we have

$$\begin{bmatrix} dz_{1,t+1} \\ dz_{2,t+1} \end{bmatrix} = J \cdot \begin{bmatrix} dz_{1,t} \\ dz_{2,t} \end{bmatrix} + WA^{-1}C \cdot d\varepsilon_{t+1}$$

$$\Leftrightarrow \begin{cases} dz_{1,t+1} = J_1 dz_{1,t} + T_1 d\varepsilon_{t+1}, \\ dz_{2,t+1} = J_2 dz_{1,t} + T_2 d\varepsilon_{t+1}. \end{cases} \tag{H.2}$$

Now, we can solve $dz_{1,t}$ and $dz_{2,t}$ as follows. From the first equation in (H.2), we can obtain

$$dz_{1,t} = -\sum_{j=1}^{\infty} J_1^{-j} T_1 d\varepsilon_{t+j}, \quad t \geq 0. \tag{H.3}$$

From equation (H.1), we can obtain

$$dz_{2,0} = [W_{22} - W_{21}W_{11}^{-1}W_{12}]dx_0 + W_{21}W_{11}^{-1}dz_{1,0}, \tag{H.4}$$

where $dx_0 = [0, dg_0, d\tilde{w}_0, d\tilde{r}_0^a]'$. From the second equation in (H.2), we have

$$dz_{2,t} = J_2 dz_{1,t-1} + T_2 d\varepsilon_t, \quad t \geq 1. \tag{H.5}$$

With the solved $dz_{1,t}$ and $dz_{2,t}$, we can solve dx_t and dy_t using equation (H.1) as follows.

$$\begin{aligned} dx_t &= [W_{22} - W_{21}W_{11}^{-1}W_{12}]^{-1}\{dz_{2,t} - W_{21}W_{11}^{-1}dz_{1,t}\}, \quad \text{and} \\ dy_t &= W_{11}^{-1}(dz_{1,t} - W_{12}dx_t). \end{aligned} \tag{H.6}$$

Once we substitute $d\varepsilon_t$ with the impulse responses of $[dg_t, d\tilde{w}_t, dr_t^a]'$ to a trend shock in equations (H.3), (H.4), (H.5), and (H.6), we obtain the impulse responses of $dy_t (= d\tilde{C}_t)$ and $dx_t (= [d\tilde{A}_{t-1}, dg_t, d\tilde{w}_t, dr_t^a]')$. The consumption response can be decomposed into the response driven by w_t and $h_t(\cdot)$ and the response driven by r_t^a by substituting $d\varepsilon_t$ with $[dg_t, d\tilde{w}_t, 0]'$ and $[0, 0, dr_t^a]'$, respectively, instead of $[dg_t, d\tilde{w}_t, dr_t^a]'$. Lastly, the response of GHH_t to a trend shock (and its decomposition) can be recovered by using the following equilibrium relationship.

$$dG\tilde{H}H_t (:= d(GHH_t / X_{t-1})) = d\tilde{C}_t - \frac{L_{ss}}{\omega} d\tilde{w}_t. \tag{H.7}$$

I Decomposition of the Consumption Variance Change from the Benchmark to the Counterfactual Economies

In section 5, I show that in the counterfactual economy, excess consumption volatility disappears in most of the posterior distribution, including the posterior mean, median, and mode. Moreover, such a change is driven by the volatility change of consumption (rather than that of income). In this section, I decompose the change of consumption variance (from the benchmark to the counterfactual economies) into the changes originating from each shock.

Let $V(\Delta \log C_t)^{\mathfrak{B}}$ be the consumption variance in the benchmark economy, and $V(\Delta \log C_t)_s^{\mathfrak{B}}$, $s \in \{z, g, \mu, \eta\}$ be its decomposed variances across shocks. Similarly, let $V(\Delta \log C_t)^{\mathfrak{C}}$ be the consumption variance in the counterfactual economy, and $V(\Delta \log C_t)_s^{\mathfrak{C}}$, $s \in \{z, g, \mu, \eta\}$ be its decomposed variances across shocks. The variance change (in ratio) is decomposed as follows.

$$\frac{V(\Delta \log C_t)^{\mathfrak{C}} - V(\Delta \log C_t)^{\mathfrak{B}}}{V(\Delta \log C_t)^{\mathfrak{B}}} = \sum_{s \in \{z, g, \mu, \eta\}} \frac{V(\Delta \log C_t)_s^{\mathfrak{C}} - V(\Delta \log C_t)_s^{\mathfrak{B}}}{V(\Delta \log C_t)^{\mathfrak{B}}}. \tag{I.1}$$

Table I.1: Decomposition of the consumption variance change

total	z	g	μ	η
-0.322	-0.180	0.137	-0.000	-0.279
(0.247)	(0.023)	(0.073)	(0.000)	(0.252)

Notes: The consumption variance change decomposition (I.1) is conducted under each posterior draw, and the mean and standard deviation (in parentheses) of each decomposed component over the posterior distribution are reported.

I conduct the variance change decomposition (I.1) under each posterior draw. Table I.1 reports the mean and standard deviation of each decomposed component over the posterior distribution. In terms of the posterior mean, consumption variance decreases by 32.2% in the counterfactual economy. Out of this -32.2% change, -27.9%p and -18.0%p come from η and z shocks generating fluctuations less, respectively, while +13.7%p comes from a g shock generating fluctuations more. As discussed in section 5, η and z shocks generate less consumption fluctuations in the counterfactual economy because households exhibit lower MPC and a correspondingly weaker precautionary saving behavior. On the other hand, a g shock generates more consumption fluctuations in the counterfactual economy because the permanent income effect the shock revives as the counteracting precautionary saving effect becomes weak.

Another noteworthy observation is that the large posterior standard deviation of the total consumption variance change inherits from that of the consumption variance change generated by an η shock. As discussed in the last paragraph of 5, a relatively small but persistent η shock and a large but transitory η shock can both generate a strong and sharp consumption response in the benchmark economy and thus are not well distinguished by a posterior likelihood in the Bayesian estimation. However, these shocks generate very different consumption responses in the counterfactual economy: the consumption response becomes much weaker in the counterfactual economy than in the benchmark economy when the shock is relatively small but persistent, while it becomes not much weaker (and sometimes becomes even stronger) when the shock is large but transitory.

J A Debate: What Drives Large Consumption Swings?

My paper is most closely related to Guntin et al. (2022). These two papers share the view that micro data, when interpreted through a heterogeneous-agent model, provide important information about what drives large consumption fluctuations. However, they come to different conclusions: Guntin et al. (2022) find that the permanent income effect of a trend shock drives large consumption swings, while I find that a financial friction shock and a stationary productivity shock mainly drive consumption fluctuations. In this sense, a long-standing debate on what drives consumption fluctuations in emerging economies, particularly between a trend shift and financial frictions[17], continues in the heterogeneous-agent open economy landscape. In this section, I discuss key differences between the two papers and how they come to different conclusions.

Micro Moments. The two papers use different information from micro data. Guntin et al. (2022) use a group-level consumption-income elasticity between the peak and trough around a crisis episode, $\hat{\varepsilon}^G_{GOP} := \frac{\log \bar{c}^G_{t+h} - \log \bar{c}^G_t}{\log \bar{y}^G_{t+h} - \log \bar{y}^G_t}$, where $\bar{c}^G_t$ and $\bar{y}^G_t$ are the group-average (residualized) consumption and income, and t and $t+h$ are the peak and trough around a crisis, respectively. On the other hand, I use an MPC out of idiosyncratic transitory income shocks obtained by using Blundell et al. (2008)'s method.

The two micro moments exhibit two important differences. First, the source of income variation is different. Guntin et al. (2022)'s elasticity washes out all the idiosyncratic income risk by group-averaging consumption and income and exploits only aggregate income risk borne by each group. In contrast, my MPC moment exploits idiosyncratic income risk only.[18] Given that individual households face much greater idiosyncratic risk than aggregate risk[19], the MPC moment may capture household consumption smoothing disruption better than the group-level elasticity.

Second, Guntin et al. (2022)'s elasticity is determined by two periods, the peak and trough around a crisis, and thus can be sensitive to timing identification. They identify the peak and trough around the 2008 Peruvian recession as 2007 and 2010, respectively, based on the aggregated individual consumption from ENAHO (rather than based on national accounts). This identification can be affected by time-varying measurement errors in the survey. Indeed, the first two panels of Figure J.1 plot the output and consumption in national accounts and suggest that the Peruvian economy was not in the trough during 2010.[20] On the other hand, my MPC moments are relatively free from the timing issue, as they are based on all periods in which data are available.

Models. Both papers interpret micro moments using a heterogeneous-agent small-open-economy model, but their models also exhibit two important differences. First, the aggregate precautionary savings correspond to liquid wealth in Guntin et al. (2022)'s model, while it corresponds to total wealth in my model.[21] As a result, households in my model exhibit much stronger precautionary saving behavior than those in Guntin et al. (2022)'s model. For this reason, a trend shock cannot

[17]For a trend shift, see Aguiar and Gopinath (2007). For financial frictions, see Neumeyer and Perri (2005), Garcia-Cicco et al. (2010), Chang and Fernández (2013), Mendoza (2010), and Bianchi (2011).

[18]The effect of aggregate income risk is removed when extracting a predictable component of income and consumption, which includes a time-fixed effect.

[19]To have a quantitative sense of their relative magnitude, one can compare the log growth dispersion between aggregate and idiosyncratic incomes. $\sigma(\Delta \log e_{i,t})$ is 0.689 in ENAHO, where $\log e_{i,t}$ is the unpredictable component of log earnings. This number is 25.2 times as large as $\sigma(\Delta \log Y_t) = 0.027$, where Y_t is aggregate income.

[20]An expert diagnosis shares this view: Economic Commission for Latin America and the Caribbean (2010) notes on page 79 that "[t]he Peruvian economy was strong in 2010, driven by growing domestic demand."

[21]To be precise, the aggregate precautionary savings correspond to the workers' share (52.2%) of the total wealth.

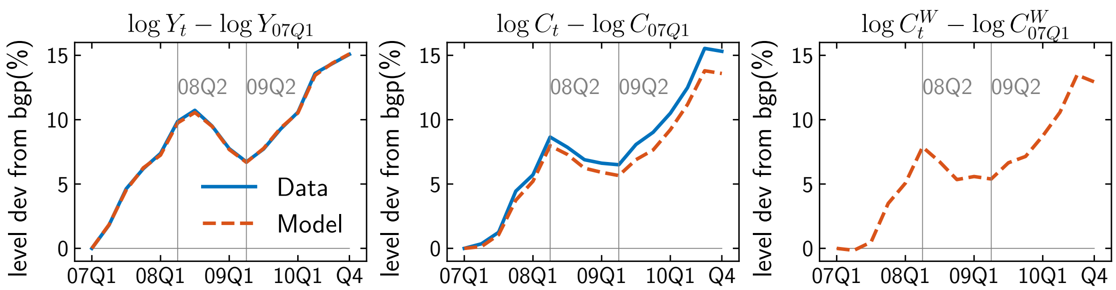


Figure J.1: What Happens in 2007–2010

Notes: The first two panels plot i) output and consumption from national accounts in logs after seasonal adjustment and log-linear detrending (labeled 'Data') and ii) their model counterparts simulated using smoothed shocks at the posterior mode (labeled 'Model'). The third panel plots the smoothed average workers' consumption.

generate large consumption fluctuations in my model, while it can in Guntin et al. (2022)'s model; a consumption response to a trend shock is muted in my model because an enhanced precautionary saving effect offsets the permanent income effect; in Guntin et al. (2022)'s model, the enhanced precautionary saving effect is far weaker, and thus, the permanent income effect can dominate.

Second, when examining whether a model can explain a micro data pattern, Guntin et al. (2022) simulate a crisis by hitting the economy with a one-time, single-type shock. In the data, they find a flat or upward-sloping graph of their elasticity over income deciles (*i.e.*, higher-income households exhibit higher elasticity). In the model, they consider two scenarios: i) a trend shock hits the economy where households face constant borrowing constraints, and ii) a stationary productivity shock hits the economy where households face aggregate-income-dependent borrowing constraints. They find that the first scenario can explain the flat or upward-sloping elasticity graph, while the second scenario cannot, as it produces a downward-sloping graph. Based on the simulation results, they conclude that a trend shock drives large consumption swings during a crisis.

My model allows different types of shocks to hit the economy at different times and, as a result, depicts a quite different story about what happened during the 2008 Peruvian recession. Importantly, I obtain an upward-sloping graph of Guntin et al. (2022)'s elasticity in the model although heightened financial friction plays a major role in the story, suggesting that the upward-sloping graph does not necessarily favor trend shift theory over financial friction theory in my model.

To see how my model depicts the 2008 Peruvian recession, I smooth aggregate shocks at the posterior mode.[22] As the first two panels of Figure J.1 show, the simulated output and consumption using smoothed shocks in the model closely track the data counterparts in 2007–2010.[23] The

[22] I thank Nils Gornemann for suggesting the smoothing analysis.

[23] Figure K.1 in Online Appendix K plots all the simulated observable variables using smoothed shocks in the

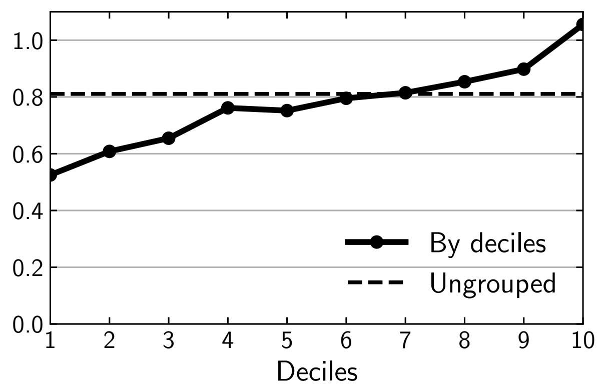


Figure J.2: Guntin et al. (2022)'s Group-Level Consumption-Income Elasticity in My Model

third panel plots smoothed workers' consumption, showing similar dynamics as total consumption. Using the smoothed shocks, I compute Guntin et al. (2022)'s elasticity around the recession. Based on the consumption and output dynamics in Figure J.1, I define the peak and trough as 2008Q2 and 2009Q2, respectively (which are indicated by gray vertical lines).[24] Figure J.2 plots the elasticity at each earnings decile, exhibiting an upward-sloping graph.

Figures J.3 and J.4 show what happened during the recession according to the simulation with smoothed shocks. As Figure J.3 shows, a large financial friction shock (η) hits the economy first in 2008Q3, while productivities (z and g) remain stable. Afterwards, large z and g shocks hit the economy in the following quarters (2008Q4 and 2009Q1, respectively). The financial friction (η) is at its peak in 2008Q4 and then comes back close to a precrisis level in 2009Q1, while productiv-

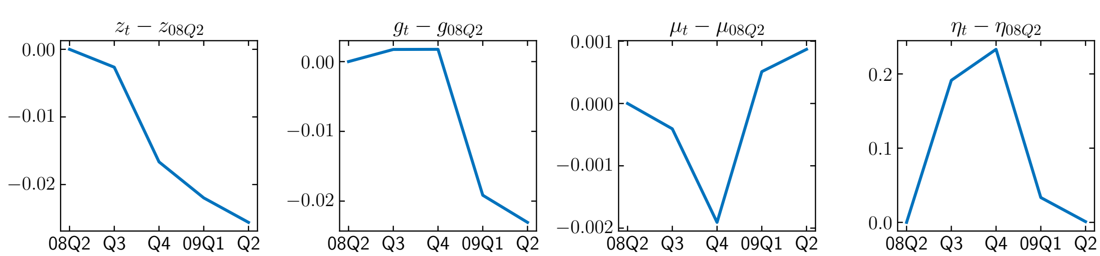


Figure J.3: Smoothed Shocks around the 2008 Peruvian Recession

model and their data counterparts throughout the data period (1980–2018), showing that the model and data track each other very closely. As explained in the Online Appendix, they are not exactly equal only because measurement errors are not included in the simulation.

[24] Additional details regarding the elasticity calculation are worth noting: i) my model is evaluated at the posterior mode; ii) the elasticity is measured for a synthetic group (*i.e.*, not for a fixed group), as in Guntin et al. (2022); iii) earnings are used as the income measure, following Guntin et al. (2022)'s treatment of the ENAHO data; and iv) the sample is composed of workers only, reflecting Guntin et al. (2022)'s sample selection, where only observations reporting positive income (which is positive earnings in ENAHO) are used.

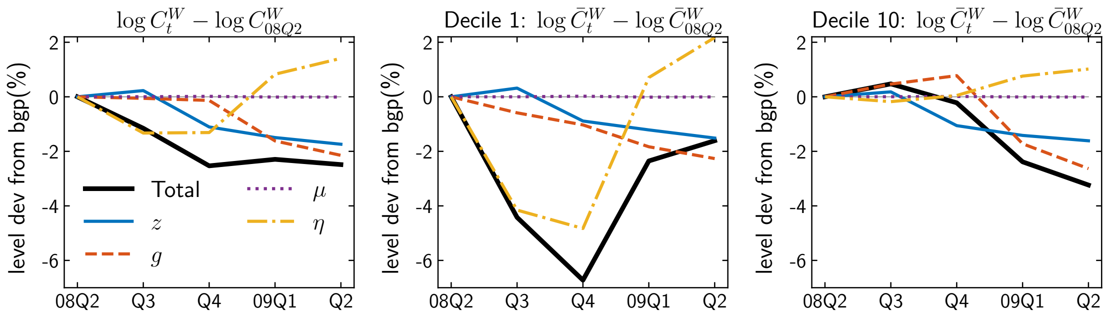


Figure J.4: Decomposition of Smoothed Workers' Consumption Fluctuations across Shocks

Notes: The three panels in Figure J.4 plot the smoothed average workers' consumption fluctuations in the whole economy and in the bottom and top earnings deciles and decompose them into fluctuations driven by each shock.

ities are still very low.[25] The first panel in Figure J.4 shows how these different shocks at different times drive consumption fluctuations during the recession. The heightened financial friction (η) drives a consumption plunge in 2008Q3. Afterwards, the stationary productivity (z) drags consumption down further in 2008Q4, and both productivities (z and g) maintain consumption at a depressed level in 2009Q1-Q2 despite alleviated financial friction. The consumption recovery due to alleviated financial friction in 2009Q1-Q2 itself goes beyond the precrisis level because of an expectation that the financial condition will be favorable for a while.

The second and third panels of Figure J.4 show how these shocks affect the group-average consumption of the bottom and top deciles differently. The heightened financial friction (η) in 2008Q3 generates a disproportionately large consumption plunge in the bottom decile, while the top decile consumption barely responds to it.[26] The consumption recovery due to alleviated financial friction in 2009Q1-Q2 is also much stronger in the bottom decile than in the top decile. The stationary productivity (z) drags consumption down during 2008Q4-2009Q2 to a similar degree between the top and bottom deciles, while the nonstationary productivity (g) drags consumption down during 2009Q1-Q2 more strongly in the top decile than in the bottom decile.[27]

In terms of a (log) consumption change between the peak (08Q2) and trough (09Q2), the bottom decile experiences a smaller change than the top decile, and this is what Guntin et al. (2022)'s elasticity captures. However, it misses a large consumption swing driven by heightened financial friction and borne disproportionately more by lower income deciles between the peak and trough.

[25] Gilchrist and Zakrajšek (2012) empirically find that a financial disruption leads a slowdown in real activities. Although their finding is for the U.S. economy, this empirical pattern is consistent with the story my model delivers about the 2008 Peruvian recession.

[26] This model behavior is in fact consistent with Guntin et al. (2022)'s model prediction that under the second scenario, where financial friction plays a major role, the elasticity graph is downward-sloping.

[27] Figure K.2 in Online Appendix K presents the decomposition of consumption fluctuations for other deciles and shows that the graphs change gradually across deciles.

K Smoothing

In Online Appendix J, I smooth aggregate shocks at the posterior mode to see how my model depicts the 2008 Peruvian recession. This section explains how the smoothing is actually implemented under the sequence space approach. This section also provides additional figures from the smoothing analysis that are omitted from Online Appendix J for brevity.

For any n-by-m matrix A, let $ravel(A)$ be an (nm)-by-1 vector defined as follows:

$$ravel(A) := \Big[[A]_1, [A]_2, \cdots, [A]_n\Big]',$$

where $[A]_i$ is the i-th row of matrix A.

Let T be the truncation length used when solving the model under the sequence space approach, and $T_c(:= 0.99T)$ be the truncation length used when evaluating model statistics.[28] Moreover, let $T_{obs}(:= 155)$ be the time length of the observed time series $[\Delta\log Y_t, \Delta\log C_t, \Delta\log I_t, \Delta TB_t/Y_t]$ during 1980Q2-2018Q4. Let $n_{exo}(:= 4)$ be the number of aggregate exogenous variables (*i.e.*, z_t, g_t, μ_t, and η_t), and $n_{obs}(:= 4)$ be the number of the observable variables (*i.e.*, $\Delta\log Y_t, \Delta\log C_t, \Delta\log I_t$, $\Delta TB_t/Y_t]$). I define an $\big(n_{exo} \times (T_c + T_{obs} - 1)\big)$-by-1 vector $\mathbf{E}$ and an $(n_{obs} \times T_{obs})$-by-1 vector $\mathbf{Y}$ as follows.

$$\mathbf{E} := ravel\left(\begin{bmatrix} \varepsilon^z_{-T_c+1}, & \varepsilon^g_{-T_c+1}, & \varepsilon^\mu_{-T_c+1}, & \varepsilon^\eta_{-T_c+1} \\ \varepsilon^z_{-T_c+2}, & \varepsilon^g_{-T_c+2}, & \varepsilon^\mu_{-T_c+2}, & \varepsilon^\eta_{-T_c+2} \\ \vdots & \vdots & \vdots & \vdots \\ \varepsilon^z_0, & \varepsilon^g_0, & \varepsilon^\mu_0, & \varepsilon^\eta_0 \\ \vdots & \vdots & \vdots & \vdots \\ \varepsilon^z_{T_{obs}-1}, & \varepsilon^g_{T_{obs}-1}, & \varepsilon^\mu_{T_{obs}-1}, & \varepsilon^\eta_{T_{obs}-1} \end{bmatrix}\right), \quad \text{and}$$

$$\mathbf{Y} := ravel\left(\begin{bmatrix} d\Delta\log Y_0, & d\Delta\log C_0, & d\Delta\log I_0, & d\Delta(TB_0/Y_0) \\ d\Delta\log Y_1, & d\Delta\log C_1, & d\Delta\log I_1, & d\Delta(TB_1/Y_1) \\ \vdots & \vdots & \vdots & \vdots \\ d\Delta\log Y_{T_{obs}-1}, & d\Delta\log C_{T_{obs}-1}, & d\Delta\log I_{T_{obs}-1}, & d\Delta(TB_{T_{obs}-1}/Y_{T_{obs}-1}) \end{bmatrix}\right),$$

where d is a demeaning operator (*i.e.*, for an observable variable OBS_t and its long-run average $\overline{OBS}_t$, $dOBS_t := OBS_t - \overline{OBS}_t$).

Because each aggregate shock is assumed to follow a normal distribution, a concatenated vector

[28] See Online Appendix E for a related discussion.

$[\mathbf{E}', \mathbf{Y}']'$ follows a multivariate normal distribution. Specifically,

$$\begin{bmatrix} \mathbf{E} \\ \mathbf{Y} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{\mathbf{EE}}, & \Sigma_{\mathbf{EY}} \\ \Sigma_{\mathbf{YE}}, & \Sigma_{\mathbf{YY}} \end{bmatrix} \right),$$

where $\Sigma_{\mathbf{EE}}$ and $\Sigma_{\mathbf{YY}}$ are the variance-covariance matrices of $\mathbf{E}$ and $\mathbf{Y}$, respectively, and $\Sigma_{\mathbf{EY}}$ and $\Sigma_{\mathbf{YE}}(=\Sigma'_{\mathbf{EY}})$ are the covariance matrices between $\mathbf{Y}$ and $\mathbf{E}$.

The relationship between $\mathbf{E}$ and $\mathbf{Y}$ can be described as follows.

$$\mathbf{Y} = \Phi \cdot \mathbf{E} + \mathbf{W}, \quad \mathbf{W} \sim N(0, \Sigma_{\mathbf{WW}}), \tag{K.1}$$

where Φ is an $(n_{obs} \times T_{obs})$-by-$\big(n_{exo} \times (T_c + T_{obs} - 1)\big)$ matrix whose elements are impulse response coefficients, and $\mathbf{W}$ is an $(n_{obs} \times T_{obs})$-by-1 vector whose elements are measurement errors assumed in the Bayesian estimation. $\Sigma_{\mathbf{WW}}$ is the variance-covariance matrix of $\mathbf{W}$.

Given the parameter values at the posterior mode, we know $\Sigma_{\mathbf{EE}}$ and $\Sigma_{\mathbf{WW}}$. By solving the model at the posterior mode, we obtain the impulse response matrix Φ. Then, using equation (K.1), we can compute $\Sigma_{\mathbf{YY}}$, $\Sigma_{\mathbf{YE}}$, and $\Sigma_{\mathbf{YE}}$ as follows.

$$\Sigma_{\mathbf{YY}} = \Phi \cdot \Sigma_{\mathbf{EE}} \cdot \Phi' + \Sigma_{\mathbf{WW}}, \quad \Sigma_{\mathbf{YE}} = \Phi \cdot \Sigma_{\mathbf{EE}}, \quad \text{and} \quad \Sigma_{\mathbf{EY}} = \Sigma'_{\mathbf{YE}}. \tag{K.2}$$

Using equation (K.2), aggregate shocks are smoothed as follows.

$$\mathbf{E}^{sm} := E[\mathbf{E}|\mathbf{Y}] = \Sigma_{\mathbf{EY}} \cdot \Sigma_{\mathbf{YY}}^{-1} \cdot \mathbf{Y} = \Sigma_{\mathbf{EE}} \cdot \Phi' \cdot (\Phi \cdot \Sigma_{\mathbf{EE}} \cdot \Phi' + \Sigma_{\mathbf{WW}})^{-1} \cdot \mathbf{Y}. \tag{K.3}$$

Using the smoothed shocks, I simulate the observable variables in the model as follows.

$$\mathbf{Y}^{sm} := \Phi \mathbf{E}^{sm} = \Phi \Sigma_{\mathbf{EE}} \cdot \Phi' \cdot (\Phi \cdot \Sigma_{\mathbf{EE}} \cdot \Phi' + \Sigma_{\mathbf{WW}})^{-1} \cdot \mathbf{Y}. \tag{K.4}$$

In Figure K.1, I plot the simulated observable variables ($\mathbf{Y}^{sm}$) and their data counterpart ($\mathbf{Y}$), showing that they track each other very closely. $\mathbf{Y}$ and $\mathbf{Y}^{sm}$ are not exactly equal only because smoothed measurement errors $E[\mathbf{W}|\mathbf{Y}]$ are not included in the simulation. Specifically,

$$\mathbf{Y} = E[\mathbf{Y}|\mathbf{Y}] = \Phi \cdot E[\mathbf{E}|\mathbf{Y}] + E[\mathbf{W}|\mathbf{Y}] = \mathbf{Y}^{sm} + E[\mathbf{W}|\mathbf{Y}].$$

$$\therefore \mathbf{Y} - \mathbf{Y}^{sm} = E[\mathbf{W}|\mathbf{Y}].$$

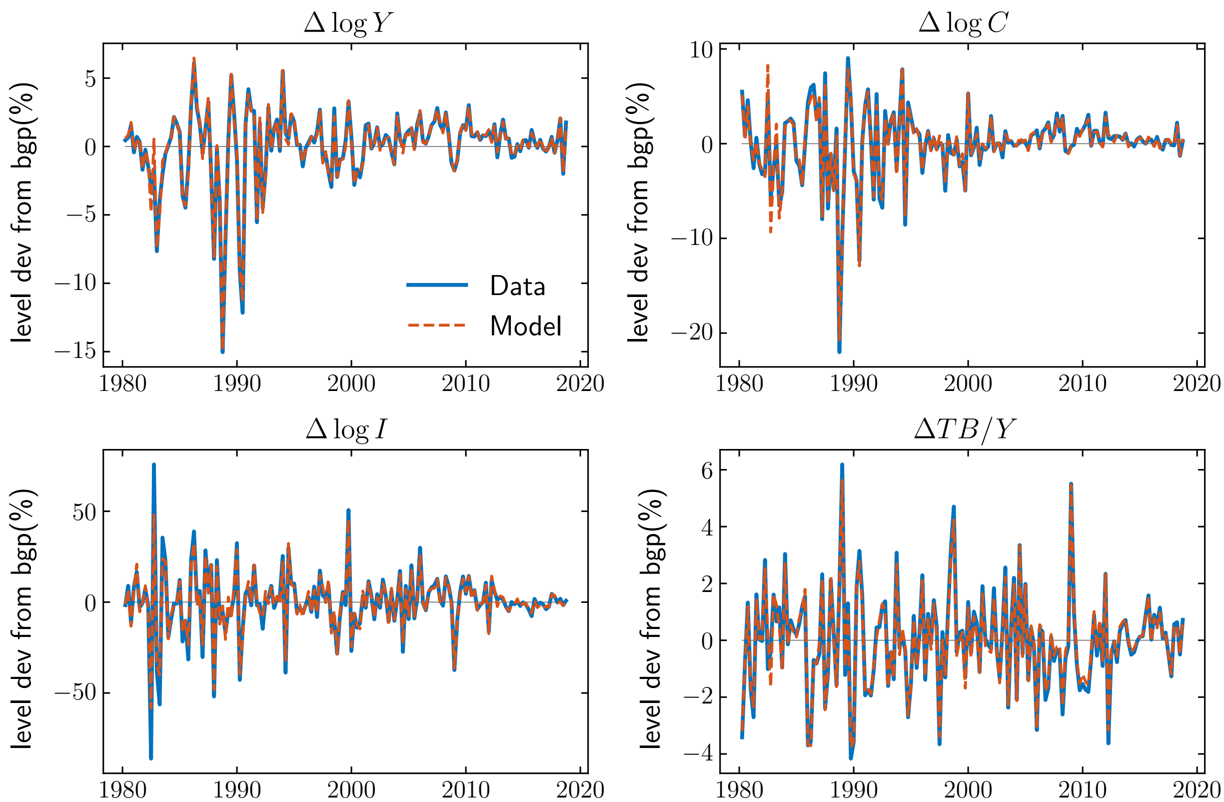


Figure K.1: Simulated Observable Variables using Smoothed Shocks

Notes: This figure plots simulated observable variables $[d\Delta \log Y_t, d\Delta \log C_t, d\Delta \log I_t, d\Delta(TB_t/Y_t)]$ using smoothed shocks (labeled 'Model') and their data counterpart (labeled 'Data').

As discussed in Online Appendix J, I simulate the consumption dynamics of workers within each earnings decile using smoothed shocks and decompose them into the dynamics generated by each shock. Figure J.4 plots the workers' consumption dynamics and their decomposition in the bottom and top deciles only. In Figure K.2 below, I plot them within each of all ten deciles. This figure shows that the graphs change gradually across deciles.

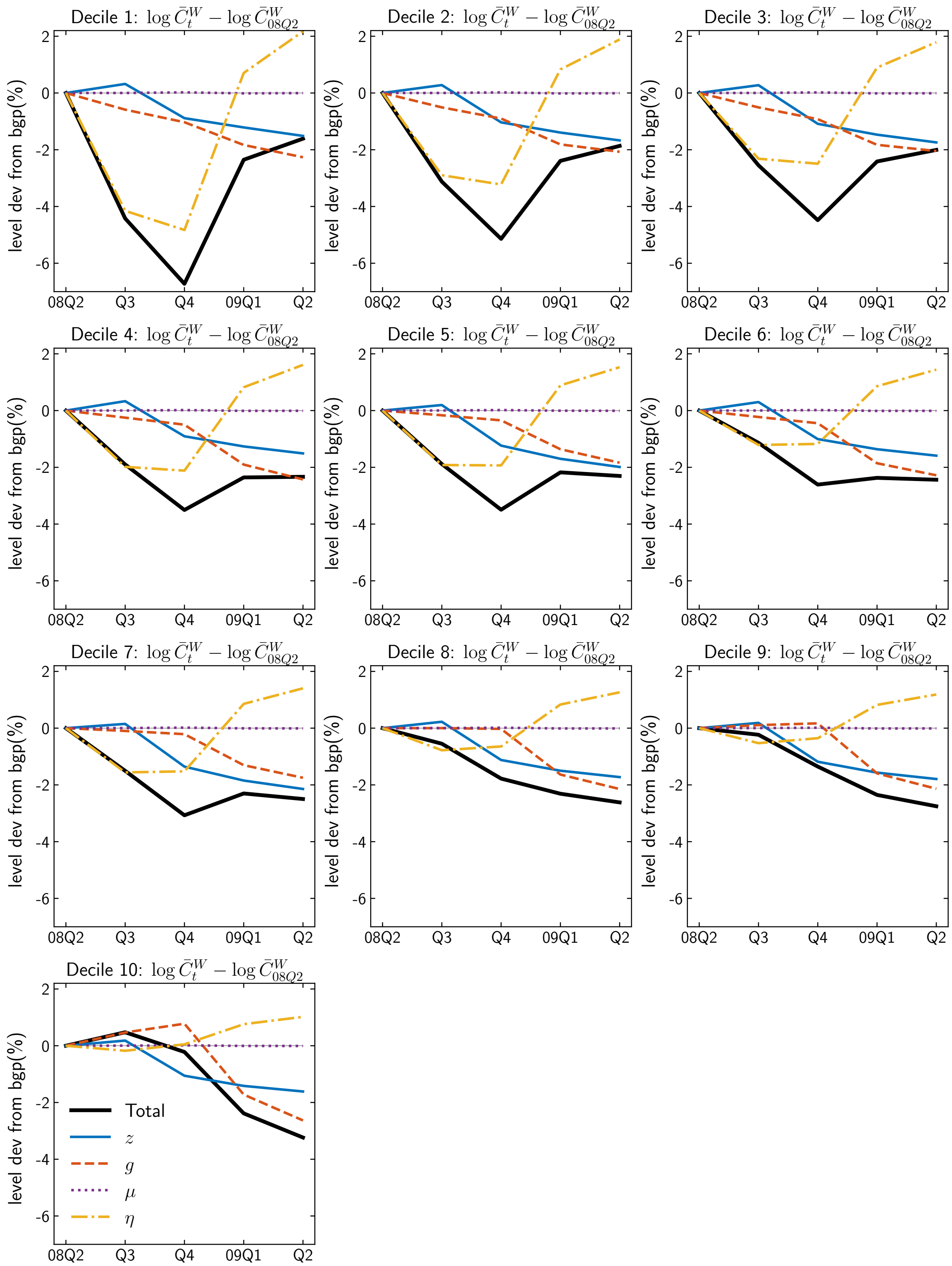


Figure K.2: Decomposition of Smoothed Workers' Consumption Fluctuations across Shocks within Each Earnings Decile

Notes: This figure plots the smoothed average workers' consumption fluctuations within each earnings decile and decomposes them into fluctuations driven by each shock.

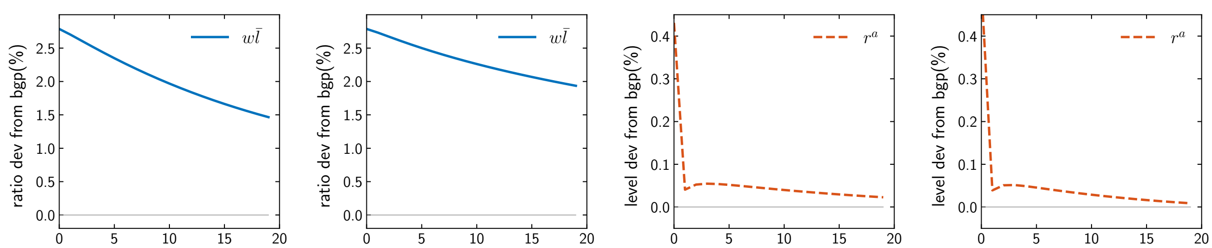
L Additional Figures

In this section, I present additional figures that are omitted in the main text for brevity.

L.1 Impulse Responses of Main Drivers to a Stationary Productivity Shock (z)

In section 5, I compare the consumption response decomposition result between the benchmark and counterfactual economies. One of the main findings is that the consumption response to a z shock is far weaker in the counterfactual economy than in the benchmark economy because the consumption responses driven by $w_t\bar{l}_t$ and r_t^a are far weaker. In principle, there are two possible channels that can yield this result: i) the drivers ($w_t\bar{l}_t$ and r_t^a) themselves might respond less to a z shock in the counterfactual economy, or ii) households might face similar fluctuations of these drivers but translate them far less into consumption fluctuations in the counterfactual economy.

To distinguish these two possible channels, in Figure L.1, I compare the impulse responses of the drivers ($w_t\bar{l}_t$ and r_t^a) to a z shock between the benchmark and counterfactual economies. Figure L.1 shows that the impulse responses of the drivers to a z shock are very similar between the benchmark and counterfactual economies, showing that it is the second channel that drives the weaker consumption response to a z shock in the counterfactual economy.



(a) $w_t\bar{l}_t$, Benchmark (b) $w_t\bar{l}_t$, Counterfactual (c) r_t^a, Benchmark (d) r_t^a, Counterfactual

Figure L.1: Impulse Responses of Main Drivers to a z Shock: Benchmark vs. Counterfactual

Notes: The impulse responses to a one-standard-deviation shock are computed at each posterior draw, and their means across the posterior distribution are plotted.

L.2 Impulse Responses of All Equilibrium Variables

In this subsection, I plot the impulse responses of all equilibrium variables to each aggregate shock in the RASOE (z, g, μ) model and the HASOE (z, g, μ, η) model.

L.2.1 RASOE (z, g, μ) Model, z Shock

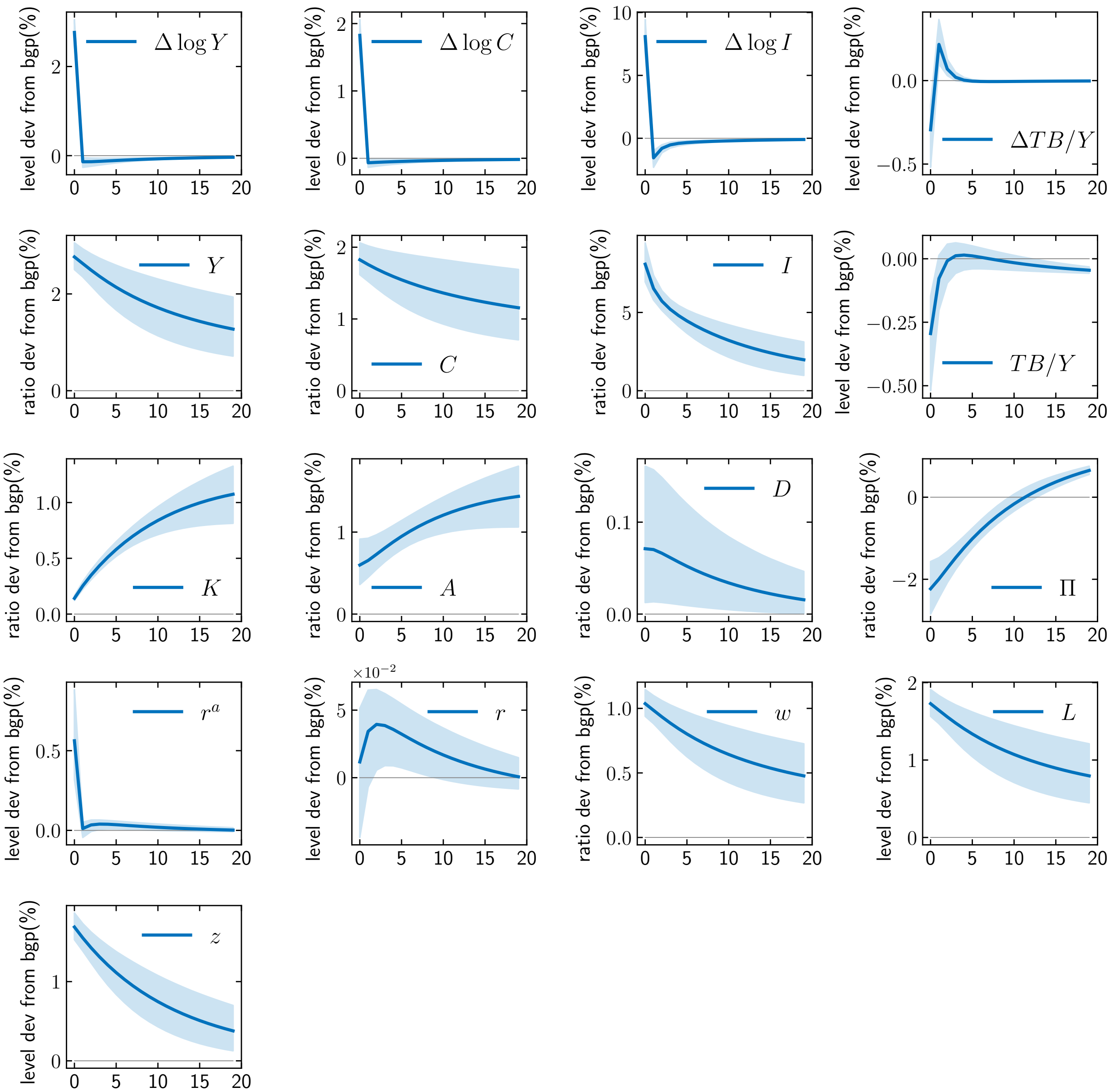


Figure L.2: Impulse Responses of All Equilibrium Variables: RASOE (z, g, μ) Model, z Shock

Notes: The impulse responses to a one-standard-deviation shock are computed at each posterior draw, and their means across the posterior distribution are plotted. Shaded areas represent 90% credible bands.

L.2.2 RASOE (z, g, μ) Model, g Shock

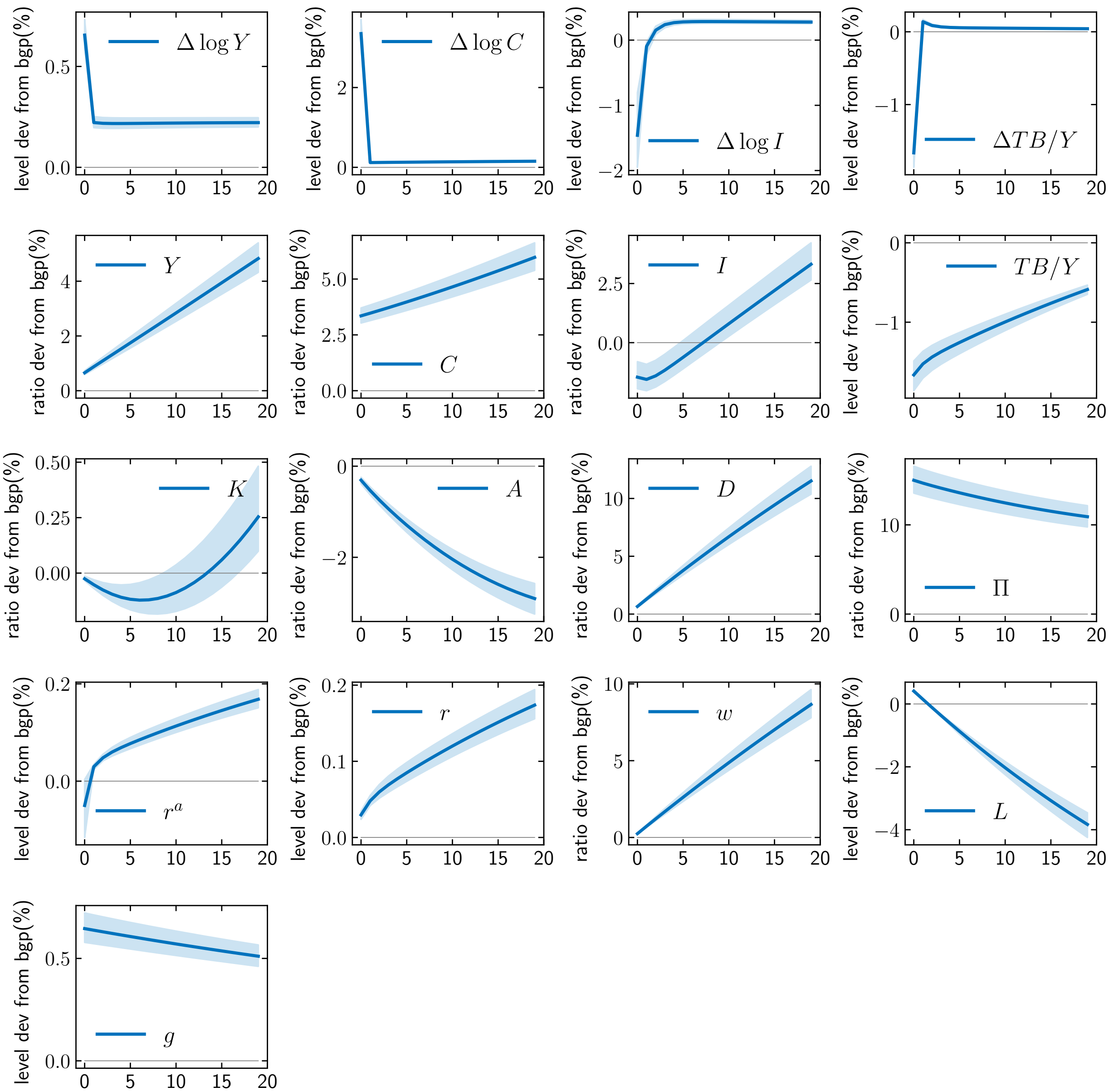


Figure L.3: Impulse Responses of All Equilibrium Variables: RASOE (z, g, μ) Model, g Shock

Notes: The impulse responses to a one-standard-deviation shock are computed at each posterior draw, and their means across the posterior distribution are plotted. Shaded areas represent 90% credible bands.

L.2.3 RASOE (z, g, μ) Model, μ Shock

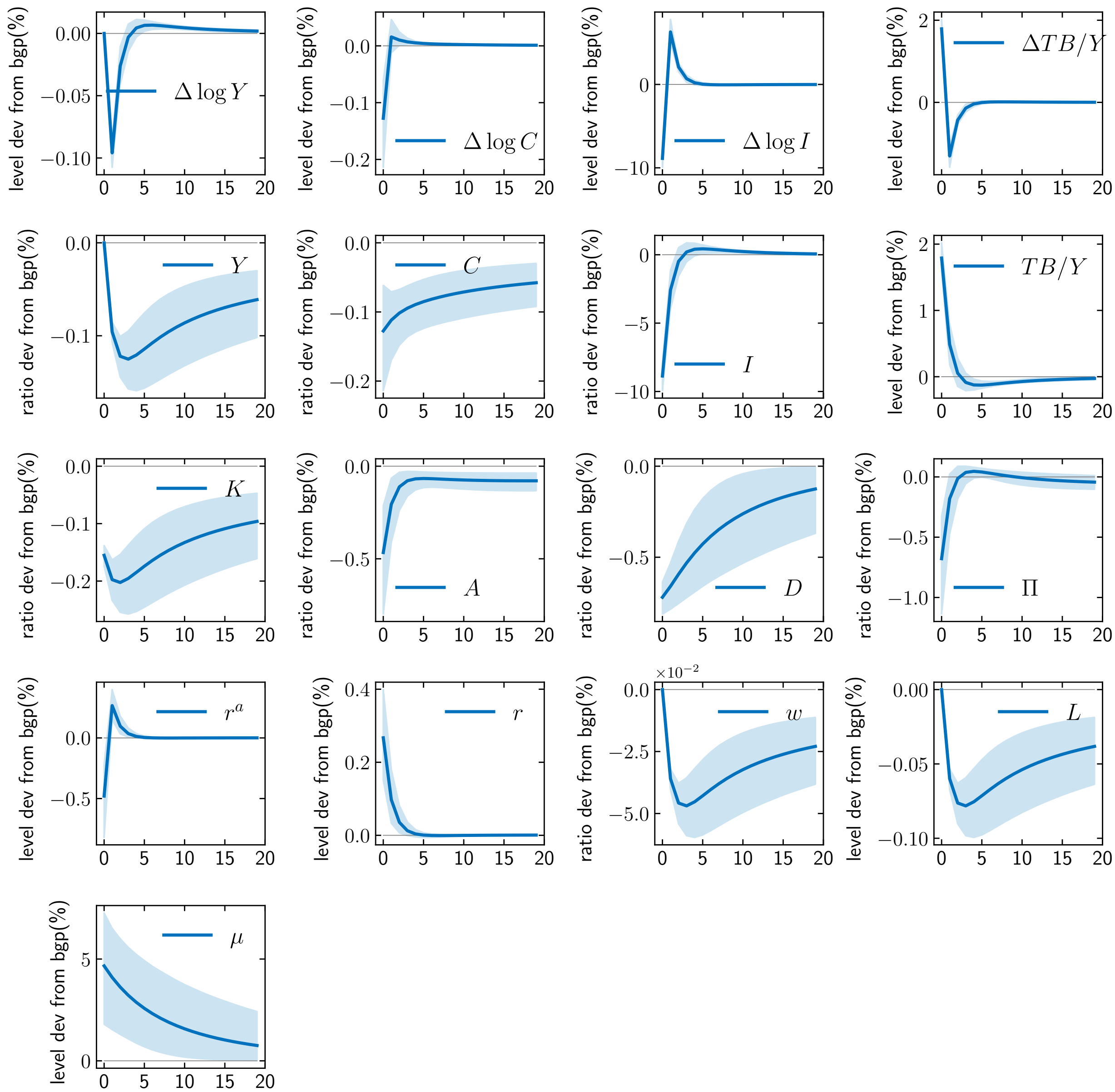


Figure L.4: Impulse Responses of All Equilibrium Variables: RASOE (z, g, μ) Model, μ Shock

Notes: The impulse responses to a one-standard-deviation shock are computed at each posterior draw, and their means across the posterior distribution are plotted. Shaded areas represent 90% credible bands.

L.2.4 HASOE (z, g, μ, η) Model, z Shock

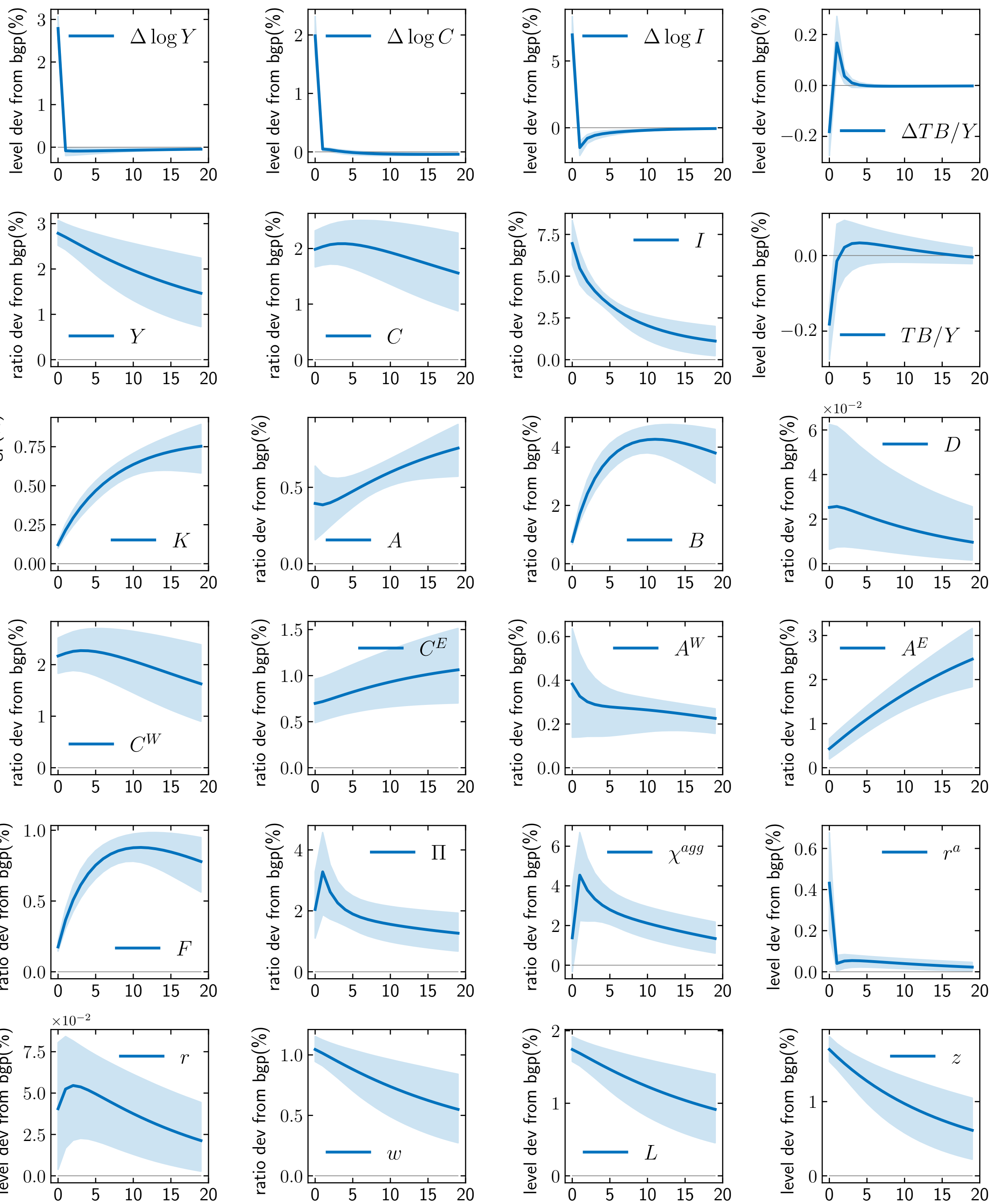


Figure L.5: Impulse Responses of All Equilibrium Variables: HASOE (z, g, μ, η) Model, z Shock

Notes: The impulse responses to a one-standard-deviation shock are computed at each posterior draw, and their means across the posterior distribution are plotted. Shaded areas represent 90% credible bands.

L.2.5 HASOE (z, g, μ, η) Model, g Shock

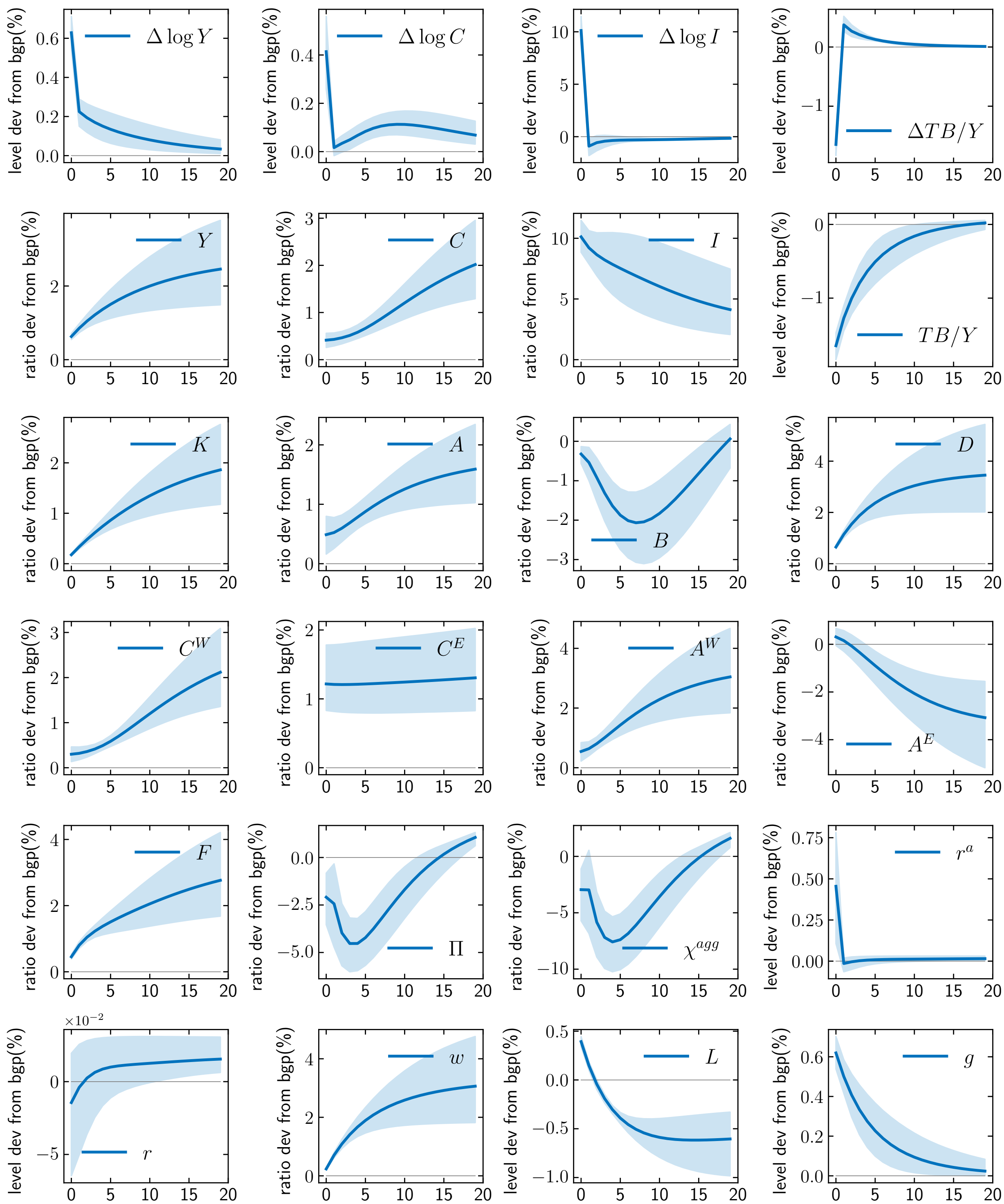


Figure L.6: Impulse Responses of All Equilibrium Variables: HASOE (z, g, μ, η) Model, g Shock

Notes: The impulse responses to a one-standard-deviation shock are computed at each posterior draw, and their means across the posterior distribution are plotted. Shaded areas represent 90% credible bands.

L.2.6 HASOE (z,g,μ,η) Model, μ Shock

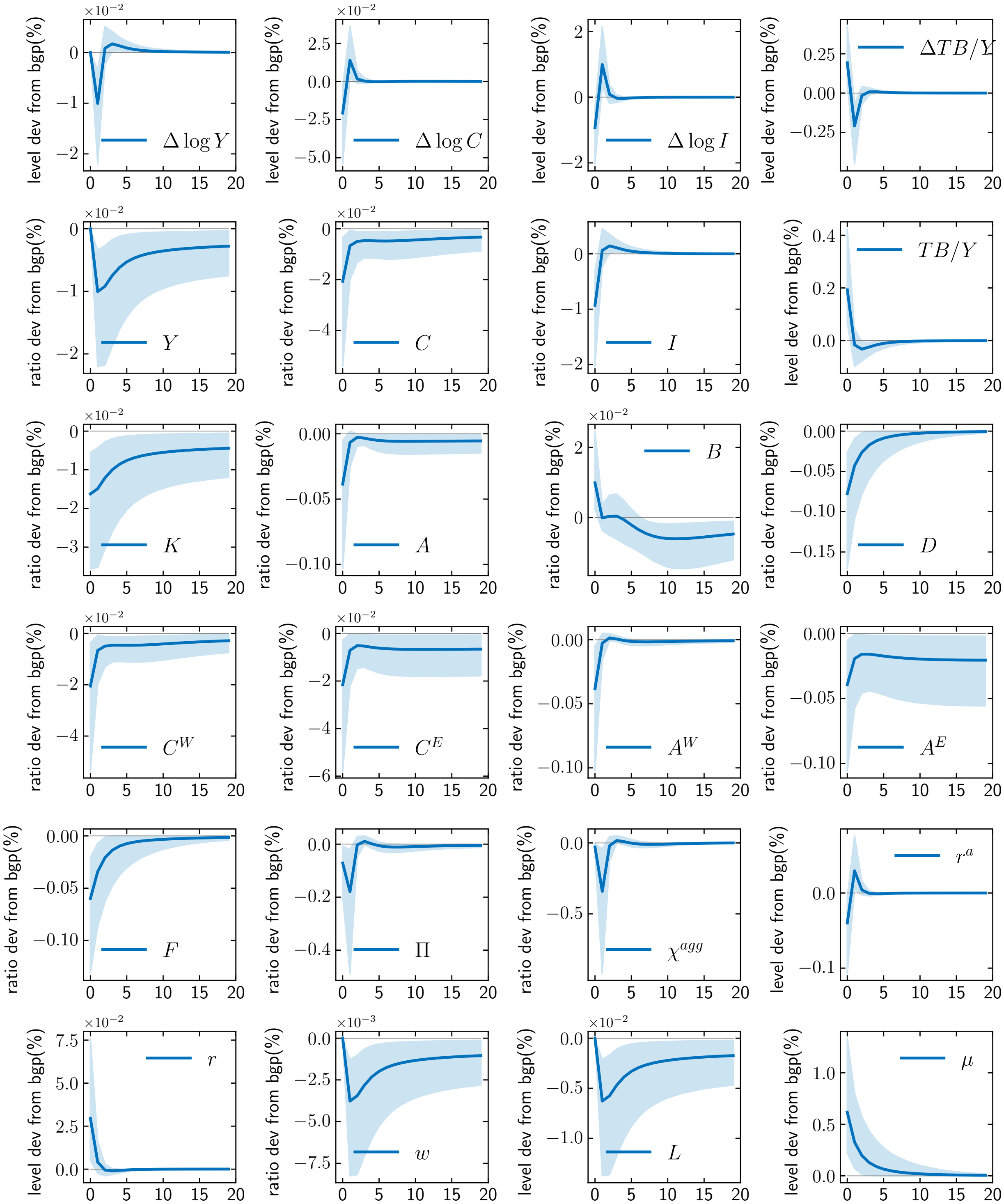


Figure L.7: Impulse Responses of All Equilibrium Variables: HASOE (z,g,μ,η) Model, μ Shock

Notes: The impulse responses to a one-standard-deviation shock are computed at each posterior draw, and their means across the posterior distribution are plotted. Shaded areas represent 90% credible bands.

L.2.7 HASOE (z,g,μ,η) Model, η Shock

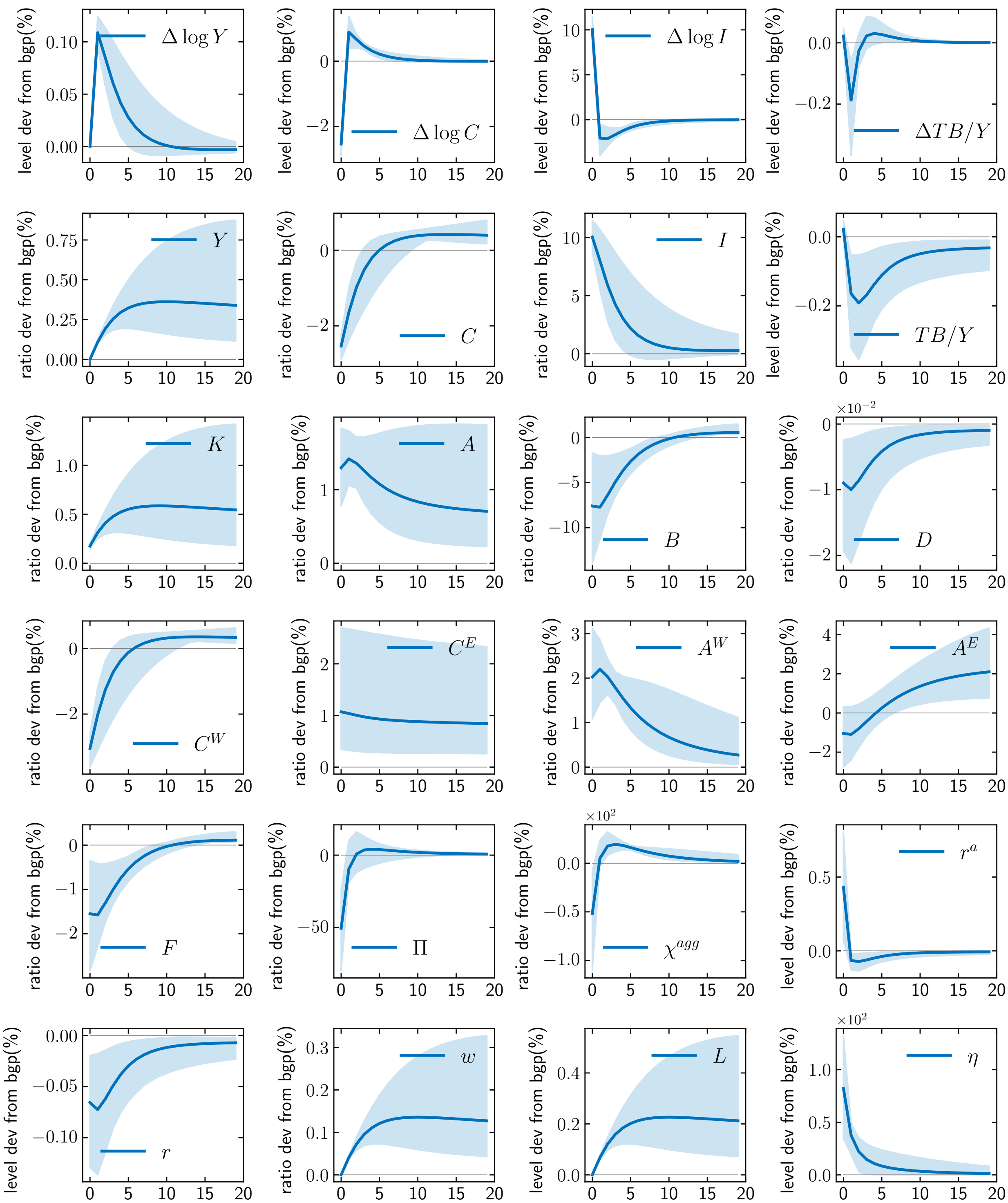


Figure L.8: Impulse Responses of All Equilibrium Variables: HASOE (z,g,μ,η) Model, η Shock

Notes: The impulse responses to a one-standard-deviation shock are computed at each posterior draw, and their means across the posterior distribution are plotted. Shaded areas represent 90% credible bands.

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