Constrained Bayesian Vector Autoregression

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This note is a companion to Bouscasse and Hong (2026). We derive analytical expressions for the likelihood, posterior distribution, and mode of the posterior distribution of a Bayesian vector autoregression (BVAR) in which some coefficients are constrained. The constraint is a linear constraint on some (perhaps all) of the autoregressive coefficients of the first equation of the BVAR. We study two cases: a flat prior and a Minnesota prior implemented with dummy variables, both with and without missing data. We use this constrained VAR in appendix E.3 of Bouscasse and Hong (2026).

1 Notations

The BVAR model is:

$$y_{t+1} = \sum_{l=1}^{L} B(l)' y_{t+1-l} + B_c' c_{t+1} + u_{t+1}, \quad u_{t+1} \sim \mathcal{N}(0, \Sigma), \tag{1}$$

where y_{t+1} is the vector of endogenous variables at time $t+1$, c_{t+1} denotes the exogenous controls, and u_{t+1} is the error term. We rewrite it by gathering the lagged values of the endogenous variables and the controls:

$$y_{t+1} = B' x_{t+1} + u_{t+1}, \quad u_{t+1} \sim \mathcal{N}(0, \Sigma), \tag{2}$$

where: $x_{t+1} = \left(y_t', y_{t-1}', \ldots, y_{t+1-L}, c_{t+1}\right)'$ and $B = (B(1)', B(2)', \ldots, B(L)', B_c')'$. Stacking the observations y_t', x_t' vertically, we can rewrite the VAR as a seemingly unrelated regressions

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system:

$$Y = XB + U, \quad U \sim \mathcal{MN}(0, I_T, \Sigma). \tag{3}$$

The dimensions of Y, X, U, B, and Σ are respectively (T, n), (T, k), (T, n), (k, n), and (n, n) where T is the number of observations, n the number of endogenous variables, k the number of right-hand side variables. Moreover, we have: $k = n \times L + q$, where L is the number of lags and q the number of control variables. $\mathcal{MN}(0, I_T, \Sigma)$ is the matrix normal distribution with variance among rows I_T and variance among columns Σ.

In our application, the vector y_t is partitioned into its debt component and its other variables: $y_t' = (d_{t-1}, y_t^{o\prime})$. We can partition B as: $B = \begin{pmatrix} B^d & B^o \end{pmatrix}$. Superscript d denotes the first column, which contains the coefficients for the debt equation; superscript o is for the $n-1$ other equations. Although we use the notation of our application, the derivations apply to any linear restriction on the coefficients of the first equation.

We write the restriction as a subset of the rows of B^d, $\mathcal{R} \subset [1, k]$, being equal to a constant column vector c:

$$B^d_{\mathcal{R}} = c. \tag{4}$$

We denote the subset of unrestricted columns of B^d, $\mathcal{U}$. Note that we have:

$$XB^d = X^{\mathcal{R}} B^d_{\mathcal{R}} + X^{\mathcal{U}} B^d_{\mathcal{U}} = X^{\mathcal{R}} c + X^{\mathcal{U}} B^d_{\mathcal{U}},$$

where $X^{\mathcal{R}}$ ($X^{\mathcal{U}}$) are the columns of X whose coefficients are restricted (unrestricted) in the debt equation. We denote $k_{\mathcal{R}}$ the number of restricted coefficients in the debt equation. Given this restriction, we note that:

$$XB = \begin{pmatrix} X^{\mathcal{R}} c + X^{\mathcal{U}} B^d_{\mathcal{U}} & XB^o \end{pmatrix} = X^{\mathcal{R}} c M^d + X^{\mathcal{U}} B^d_{\mathcal{U}} M^d + XB^o M^o, \tag{5}$$

where:

$$M^d = \begin{pmatrix} 1 & (0)_{1,n-1} \end{pmatrix}, \qquad M^o = \begin{pmatrix} (0)_{n-1,1} & I_{n-1} \end{pmatrix}.$$

2 Likelihood

The likelihood of the system described in equation (3) is:

$$p(Y,X \mid B,\Sigma) \propto |\Sigma|^{-T/2} \exp\left(-\frac{1}{2}\text{vec}\,(Y-XB)' \left(\Sigma^{-1} \otimes I_T\right) \text{vec}\,(Y-XB)\right).$$

We can rewrite $\text{vec}\,(Y-XB)$ with equation (5):

$$\text{vec}\,(Y-XB) = \text{vec}\left(Y - X^R c M^d - X^U B^d_{\mathcal{U}} M^d - X B^o M^o\right) = \mathcal{Y} - \mathcal{X}\mathcal{B},$$

where curly types denote:

$$\mathcal{Y} = \text{vec}\left(Y - X^R c M^d\right), \qquad \mathcal{X} = \begin{pmatrix} X^{\mathcal{U}} & (0) \\ (0) & I_{n-1} \otimes X \end{pmatrix}, \quad \mathcal{B} = \text{vec}\left(B \setminus B^d_{\mathcal{R}}\right).$$

So, we obtain the likelihood as:

$$p(Y,X \mid B,\Sigma) \propto |\Sigma|^{-T/2} \exp\left(-\frac{1}{2}\left(\mathcal{Y} - \mathcal{X}\mathcal{B}\right)' \left(\Sigma^{-1} \otimes I_T\right) \left(\mathcal{Y} - \mathcal{X}\mathcal{B}\right)\right). \tag{6}$$

2.1 Density for $\mathcal{B}$

We turn equation (6) into a density for $\mathcal{B}$. To do so, we transform the trace term in equation (6) by completing the square:

$$\begin{aligned} &\left(\mathcal{Y} - \mathcal{X}\mathcal{B}\right)' \left(\Sigma^{-1} \otimes I_T\right) \left(\mathcal{Y} - \mathcal{X}\mathcal{B}\right) \\ &\quad = \left(\mathcal{B} - \tilde{\mathcal{B}}\right)' \tilde{\Omega}^{-1} \left(\mathcal{B} - \tilde{\mathcal{B}}\right) + \mathcal{Y}' \left(\Sigma^{-1} \otimes I_T\right) \mathcal{Y} - \tilde{\mathcal{B}}' \tilde{\Omega}^{-1} \tilde{\mathcal{B}}, \end{aligned}$$

where:

$$\begin{aligned} \tilde{\mathcal{B}} &= \left(\mathcal{X}' \left(\Sigma^{-1} \otimes I_T\right) \mathcal{X}\right)^{-1} \left(\mathcal{X}' \left(\Sigma^{-1} \otimes I_T\right) \mathcal{Y}\right), \\ \tilde{\Omega} &= \left(\mathcal{X}' \left(\Sigma^{-1} \otimes I_T\right) \mathcal{X}\right)^{-1}. \end{aligned}$$

So, we can rewrite equation (6) by transforming the term featuring $\mathcal{B}$ into a multivariate normal density:

$$\begin{aligned} p(B,\Sigma \mid Y,X) \propto \left|\tilde{\Omega}\right|^{1/2} \mathcal{N}\left(\tilde{\mathcal{B}}, \tilde{\Omega}; \mathcal{B}\right) \\ \times |\Sigma|^{-T/2} \exp\left(-\frac{1}{2}\left(\mathcal{Y}' \left(\Sigma^{-1} \otimes I_T\right) \mathcal{Y} - \tilde{\mathcal{B}}' \tilde{\Omega}^{-1} \tilde{\mathcal{B}}\right)\right), \end{aligned} \tag{7}$$

where $\mathcal{N}\left(\tilde{\mathcal{B}},\tilde{\Omega};B\right)$ is a multivariate normal density for $\mathcal{B}$ with mean $\tilde{\mathcal{B}}$ and variance $\tilde{\Omega}$.

We need an explicit expression for $\left|\tilde{\Omega}\right|$. We first note that we can rewrite $\mathcal{X}$ as:

$$\mathcal{X} = \begin{pmatrix} X^{\mathcal{U}} & (0) \\ (0) & I_{n-1}\otimes X \end{pmatrix} = \left(I_n \otimes X\right) F, \qquad F = \begin{pmatrix} I_{\mathcal{U}} & \mathbf{0}_{(k,(n-1)k)} \\ \mathbf{0}_{((n-1)k,k-k_{\mathcal{R}})} & I_{(n-1)k} \end{pmatrix},$$

where $I_{\mathcal{U}}$ is a $(k, k-k_{\mathcal{R}})$ matrix that selects the elements of X that are unrestricted in the debt equation: $X^{\mathcal{U}} = XI_{\mathcal{U}}$. Using the definition of $\tilde{\Omega}$, we obtain $\tilde{\Omega}^{-1}$:

$$\tilde{\Omega}^{-1} = F'\left(\Sigma^{-1}\otimes(X'X)\right)F = \begin{pmatrix} (\Sigma^{-1})_{dd}\otimes(I'_{\mathcal{U}}X'XI_{\mathcal{U}}) & (\Sigma^{-1})_{do}\otimes(I'_{\mathcal{U}}X'X) \\ (\Sigma^{-1})_{od}\otimes(X'XI_{\mathcal{U}}) & \left(\Sigma^{-1}\right)_{oo}\otimes X'X \end{pmatrix}. \tag{8}$$

Equation (8) allows us to compute the determinant of $\tilde{\Omega}^{-1}$:

$$\begin{aligned}\left|\tilde{\Omega}^{-1}\right| &= \left|\left(\Sigma^{-1}\right)_{oo}\otimes X'X\right| \\ &\quad\times\Big|(\Sigma^{-1})_{dd}\otimes(I'_{\mathcal{U}}X'XI_{\mathcal{U}}) \\ &\qquad -\left((\Sigma^{-1})_{do}\left(\Sigma^{-1}\right)_{oo}^{-1}(\Sigma^{-1})_{od}\right)\otimes\left(I'_{\mathcal{U}}X'X(X'X)^{-1}X'XI_{\mathcal{U}}\right)\Big| \\ &= \left|\left(\Sigma^{-1}\right)_{oo}\right|^{k}\left|X'X\right|^{n-1}\times\left|\Sigma_{dd}^{-1}\right|^{k-k_{\mathcal{R}}}\left|I'_{\mathcal{U}}X'XI_{\mathcal{U}}\right|. \end{aligned} \tag{9}$$

It will also be useful to transform the determinant of Σ:

$$|\Sigma| = |\Sigma_{dd}|\left|\Sigma_{oo}-\Sigma_{od}\Sigma_{dd}^{-1}\Sigma_{do}\right| = |\Sigma_{dd}|\left|\left(\Sigma^{-1}\right)_{oo}^{-1}\right|. \tag{10}$$

Using equations (9–10), we can rewrite equation (7):

$$\begin{aligned} p(B,\Sigma\mid Y,X) &\propto \mathcal{N}\left(\tilde{\mathcal{B}},\tilde{\Omega};B\right) \\ &\quad\times|\Sigma_{dd}|^{-(T-(k-k_{\mathcal{R}}))/2}\left|\left(\Sigma^{-1}\right)_{oo}^{-1}\right|^{-(T-k)/2}\exp\left(-\frac{1}{2}\left(\mathcal{Y}'\left(\Sigma^{-1}\otimes I_T\right)\mathcal{Y}-\tilde{\mathcal{B}}'\tilde{\Omega}^{-1}\tilde{\mathcal{B}}\right)\right). \end{aligned} \tag{11}$$

2.2 Density for Σ

We turn equation (11) into a density for Σ. To do so, we find an expression for $\bar{\mathcal{B}}'\tilde{\Omega}^{-1}\bar{\mathcal{B}}$, which appears in equation (7). Inverting $\tilde{\Omega}^{-1}$ with standard block matrix formulas, we obtain the 4 blocks of $\tilde{\Omega}$:

$$\begin{aligned}\tilde{\Omega}_{dd} &= \Sigma_{dd}\otimes\left(I'_{\mathcal{U}}X'XI_{\mathcal{U}}\right)^{-1}, \\ \tilde{\Omega}_{oo} &= \left(\Sigma^{-1}\right)_{oo}^{-1}\otimes(X'X)^{-1}+\left(\Sigma_{od}\Sigma_{dd}^{-1}\Sigma_{do}\right)\otimes\left(I_{\mathcal{U}}\left(I'_{\mathcal{U}}X'XI_{\mathcal{U}}\right)^{-1}I'_{\mathcal{U}}\right),\end{aligned}$$

$$\tilde{\Omega}_{do} = \Sigma_{do} \otimes \left(\left(I_{\mathcal{U}}' X' X I_{\mathcal{U}} \right)^{-1} I_{\mathcal{U}}' \right),$$
$$\tilde{\Omega}_{od} = \Sigma_{od} \otimes \left(I_{\mathcal{U}} \left(I_{\mathcal{U}}' X' X I_{\mathcal{U}} \right)^{-1} \right).$$

It will soon be convenient to also know $F\tilde{\Omega}F'$:

$$\begin{aligned}
\left(F\tilde{\Omega}F'\right)_{dd} &= \Sigma_{dd} \otimes \left(I_{\mathcal{U}} \left(I_{\mathcal{U}}' X' X I_{\mathcal{U}} \right)^{-1} I_{\mathcal{U}}' \right), \\
\left(F\tilde{\Omega}F'\right)_{oo} &= \Sigma_{oo} \otimes \left(I_{\mathcal{U}} \left(I_{\mathcal{U}}' X' X I_{\mathcal{U}} \right)^{-1} I_{\mathcal{U}}' \right) + \left(\Sigma^{-1}\right)_{oo}^{-1} \otimes \left((X'X)^{-1} - \left(I_{\mathcal{U}}' X' X I_{\mathcal{U}} \right)^{-1} I_{\mathcal{U}}' \right), \\
\left(F\tilde{\Omega}F'\right)_{do} &= \Sigma_{do} \otimes \left(I_{\mathcal{U}} \left(I_{\mathcal{U}}' X' X I_{\mathcal{U}} \right)^{-1} I_{\mathcal{U}}' \right), \\
\left(F\tilde{\Omega}F'\right)_{do} &= \Sigma_{od} \otimes \left(I_{\mathcal{U}} \left(I_{\mathcal{U}}' X' X I_{\mathcal{U}} \right)^{-1} I_{\mathcal{U}}' \right),
\end{aligned}$$

where we have used: $\Sigma_{oo} = \left(\Sigma^{-1}\right)_{oo}^{-1} + \Sigma_{od}\Sigma_{dd}^{-1}\Sigma_{do}$. We can express the latter more simply:

$$F\tilde{\Omega}F' = \Sigma \otimes \Omega_{\mathcal{U}} + \tilde{\Sigma}_{oo} \otimes \left(\Omega_X - \Omega_{\mathcal{U}} \right), \tag{12}$$

where:

$$\tilde{\Sigma}_{oo} = \begin{pmatrix} 0 & (0) \\ (0) & \left(\Sigma^{-1}\right)_{oo}^{-1} \end{pmatrix}, \quad \Omega_X = (X'X)^{-1}, \quad \Omega_{\mathcal{U}} = I_{\mathcal{U}} \left(I_{\mathcal{U}}' X' X I_{\mathcal{U}} \right)^{-1} I_{\mathcal{U}}'.$$

Back to $\bar{\mathcal{B}}'\tilde{\Omega}^{-1}\bar{\mathcal{B}}$:

$$\begin{aligned}
\bar{\mathcal{B}}'\tilde{\Omega}^{-1}\bar{\mathcal{B}} &= \mathcal{Y}' \left(\Sigma^{-1} \otimes X \right) F\tilde{\Omega}F' \left(\Sigma^{-1} \otimes X' \right) \mathcal{Y} \\
&= \mathcal{Y}' \left(\Sigma^{-1} \otimes \left(X \Omega_{\mathcal{U}} X' \right) + \left(\Sigma^{-1} \tilde{\Sigma}_{oo} \Sigma^{-1} \right) \otimes \left(X (\Omega_X - \Omega_{\mathcal{U}}) X' \right) \right) \mathcal{Y}.
\end{aligned}$$

We notice that:

$$\Sigma^{-1}\tilde{\Sigma}_{oo}\Sigma^{-1} = \begin{pmatrix} \left(\Sigma^{-1}\right)_{do} \left(\Sigma^{-1}\right)_{oo}^{-1} \left(\Sigma^{-1}\right)_{od} & \left(\Sigma^{-1}\right)_{do} \\ \left(\Sigma^{-1}\right)_{od} & \left(\Sigma^{-1}\right)_{oo} \end{pmatrix} = \Sigma^{-1} - \tilde{\Sigma}_{dd.1}, \qquad \tilde{\Sigma}_{dd.1} = \begin{pmatrix} \Sigma_{dd}^{-1} & (0) \\ (0) & (0) \end{pmatrix}.$$

Finally, we have:

$$\begin{aligned}
\bar{\mathcal{B}}'\tilde{\Omega}^{-1}\bar{\mathcal{B}} = &\operatorname{tr}\left(\Sigma^{-1} Z' X (X'X)^{-1} X' Z \right) \\
&- \operatorname{tr}\left(\Sigma_{dd}^{-1} Z^d X \left((X'X)^{-1} - I_{\mathcal{U}} \left(I_{\mathcal{U}}' X' X I_{\mathcal{U}} \right)^{-1} I_{\mathcal{U}}' \right) X' Z^d \right),
\end{aligned} \tag{13}$$

where $Z = Y - X^R c M^d$ and Z^d is the debt column of Z. So, equation (11) becomes:

$$p(Y, X \mid B, \Sigma) \propto \mathcal{N}\left(\tilde{\mathcal{B}}, \tilde{\Omega}; B\right) \times |\Sigma_{dd}|^{-(T-(k-k_{\mathcal{R}}))/2} \left|\left(\Sigma^{-1}\right)_{oo}^{-1}\right|^{-(T-k)/2} \times \exp\left(-\frac{1}{2}\left(\mathrm{tr}\left(\Sigma^{-1}\tilde{S}\right) + \mathrm{tr}\left(\Sigma_{dd}^{-1}\tilde{R}_{dd}\right)\right)\right), \quad (14)$$

where:

$$\tilde{S} = Z'Z - Z'X(X'X)^{-1}X'Z,$$
$$\tilde{R}_{dd} = Z^{d'}X\left((X'X)^{-1} - I_{\mathcal{U}}\left(I_{\mathcal{U}}'X'XI_{\mathcal{U}}\right)^{-1}I_{\mathcal{U}}'\right)X'Z^{d}.$$

2.3 Change of Variables

Once we introduce the prior distributions, we perform the following change of variables: $\Sigma_{do} \rightarrow \Sigma_{dd}^{-1}\Sigma_{do}$ and $\Sigma_{oo} \rightarrow \left(\Sigma^{-1}\right)_{oo}^{-1} = \Sigma_{oo} - \Sigma_{od}\Sigma_{dd}^{-1}\Sigma_{do}$.[1] So, we rewrite the trace terms of equation (11) in terms of these new variables:

$$\mathrm{tr}\left(\Sigma^{-1}\tilde{S}\right) + \mathrm{tr}\left(\Sigma_{dd}^{-1}\tilde{R}_{dd}\right) = \mathrm{tr}\left(\Sigma_{dd}^{-1}\left(\tilde{S}_{dd} + \tilde{R}_{dd}\right)\right) + \mathrm{tr}\left(\left(\Sigma^{-1}\right)_{oo}\left(\tilde{S}^{-1}\right)_{oo}^{-1}\right) + \mathrm{tr}\left(\left(\Sigma^{-1}\right)_{oo}\left(\Sigma_{dd}^{-1}\Sigma_{do} - \tilde{S}_{dd}^{-1}\tilde{S}_{do}\right)'\tilde{S}_{dd}\left(\Sigma_{dd}^{-1}\Sigma_{do} - \tilde{S}_{dd}^{-1}\tilde{S}_{do}\right)\right).$$

We obtain a final expression for the likelihood:

$$\begin{aligned}
&p\left(Y, X \mid B, \Sigma_{dd}, \left(\Sigma^{-1}\right)_{oo}^{-1}, \Sigma_{dd}^{-1}\Sigma_{do}\right) \\
&\propto \mathcal{N}\left(\tilde{\mathcal{B}}, \tilde{\Omega}; \mathcal{B}\right) \\
&\times \left|\left(\Sigma^{-1}\right)_{oo}^{-1}\right|^{-1/2} \\
&\times \exp\left(-\frac{1}{2}\mathrm{tr}\left(\left(\Sigma^{-1}\right)_{oo}\left(\Sigma_{dd}^{-1}\Sigma_{do} - \tilde{S}_{dd}^{-1}\tilde{S}_{do}\right)'\tilde{S}_{dd}\left(\Sigma_{dd}^{-1}\Sigma_{do} - \tilde{S}_{dd}^{-1}\tilde{S}_{do}\right)\right)\right) \\
&\times |\Sigma_{dd}|^{-(T-(k-k_{\mathcal{R}}))/2} \exp\left(-\frac{1}{2}\mathrm{tr}\left(\Sigma_{dd}^{-1}\left(\tilde{S}_{dd} + \tilde{R}_{dd}\right)\right)\right) \\
&\times \left|\left(\Sigma^{-1}\right)_{oo}^{-1}\right|^{-(T-k-1)/2} \exp\left(-\frac{1}{2}\mathrm{tr}\left(\left(\Sigma^{-1}\right)_{oo}\left(\tilde{S}^{-1}\right)_{oo}^{-1}\right); \Sigma_{dd}^{-1}\Sigma_{do}\right). \quad (15)
\end{aligned}$$

[1]This change of variables is a standard step to derive the marginal distributions of a Wishart or inverse Wishart distribution. See Gupta and Nagar (2018, theorem 3.3.9) for an example.

3 Posterior Distribution

3.1 Jeffreys's Prior

We now assume Jeffreys's prior:

$$p(B, \Sigma) \propto |\Sigma|^{-(n+1)/2} = |\Sigma_{dd}|^{-(n+1)/2} \left| \left(\Sigma^{-1}\right)_{oo}^{-1} \right|^{-(n+1)/2}. \tag{16}$$

Combining this prior and the likelihood in (15):

$$\begin{aligned}
p\left(B, \Sigma \mid Y, X\right) \propto\, & p\left(Y, X \mid B, \Sigma\right) \times p(B, \Sigma) \\
\propto\, & \mathcal{N}\left(\tilde{\mathcal{B}}, \tilde{\Omega}; \mathcal{B}\right) \\
& \times \left| \left(\Sigma^{-1}\right)_{oo}^{-1} \right|^{-1/2} \\
& \times \exp\left(-\frac{1}{2}\mathrm{tr}\left(\left(\Sigma^{-1}\right)_{oo}\left(\Sigma_{dd}^{-1}\Sigma_{do} - \tilde{S}_{dd}^{-1}\tilde{S}_{do}\right)' \tilde{S}_{dd}\left(\Sigma_{dd}^{-1}\Sigma_{do} - \tilde{S}_{dd}^{-1}\tilde{S}_{do}\right)\right)\right) \\
& \times |\Sigma_{dd}|^{-(T-(k-k_{\mathcal{R}})+n+1)/2} \exp\left(-\frac{1}{2}\mathrm{tr}\left(\Sigma_{dd}^{-1}\left(\tilde{S}_{dd} + \tilde{R}_{dd}\right)\right)\right) \\
& \times \left| \left(\Sigma^{-1}\right)_{oo}^{-1} \right|^{-(T-k+n)/2} \exp\left(-\frac{1}{2}\mathrm{tr}\left(\left(\Sigma^{-1}\right)_{oo}\left(\tilde{S}^{-1}\right)_{oo}^{-1}\right); \Sigma_{dd}^{-1}\Sigma_{do}\right).
\end{aligned}$$

We now perform the change of variables mentioned in section 2: $\Sigma_{do} \to \Sigma_{dd}^{-1}\Sigma_{do}$ and $\Sigma_{oo} \to \left(\Sigma^{-1}\right)_{oo}^{-1} = \Sigma_{oo} - \Sigma_{od}\Sigma_{dd}^{-1}\Sigma_{do}$. We need to multiply the density by the determinant of the Jacobian of this change of variables, $|\Sigma_{dd}|^{n-1}$. So, we obtain:

$$\begin{aligned}
p\left(B, \Sigma_{do}, \left(\Sigma^{-1}\right)_{oo}^{-1}, \Sigma_{dd} \mid Y, X\right) & \propto \mathcal{N}\left(\tilde{\mathcal{B}}, \tilde{\Omega}; \mathcal{B}\right) \times \mathcal{MN}\left(\tilde{S}_{dd}^{-1}\tilde{S}_{do}, \tilde{S}_{dd}^{-1}, \left(\Sigma^{-1}\right)_{oo}^{-1}; \Sigma_{dd}^{-1}\Sigma_{do}\right) \\
& \times \mathcal{W}^{-1}\left(\left(\tilde{S}^{-1}\right)_{oo}^{-1}, T-k; \left(\Sigma^{-1}\right)_{oo}^{-1}\right) \times \mathcal{W}^{-1}\left(\tilde{S}_{dd} + \tilde{R}_{dd}, T-(k-k_{\mathcal{R}})-(n-1); \Sigma_{dd}\right),
\end{aligned}$$

where $\mathcal{MN}(.)$ is the density of the matrix normal distribution and $\mathcal{W}^{-1}(.)$ that of the inverse Wishart.

Proposition 1 (Jeffreys's prior) *With Jeffreys's prior (16), the posterior distributions of B and Σ are:*

$$\begin{aligned}
& \mathcal{B} \mid \Sigma, Y, X \sim \mathcal{N}\left(\tilde{\mathcal{B}}, \tilde{\Omega}\right), \\
& \Sigma_{dd}^{-1}\Sigma_{do} \mid \left(\Sigma^{-1}\right)_{oo}^{-1}, Y, X \sim \mathcal{MN}\left(\tilde{S}_{dd}^{-1}\tilde{S}_{do}, \tilde{S}_{dd}^{-1}, \left(\Sigma^{-1}\right)_{oo}^{-1}\right), \\
& \left(\Sigma^{-1}\right)_{oo}^{-1} \mid Y, X \sim \mathcal{W}^{-1}\left(\left(\tilde{S}^{-1}\right)_{oo}^{-1}, \tilde{\nu}\right),
\end{aligned}$$

$$\Sigma_{dd} \mid Y, X \sim \mathcal{W}^{-1}\left(\tilde{S}_{dd} + \tilde{R}_{dd}, \tilde{\nu} + k_{\mathcal{R}} - (n-1)\right),$$

where:

$$\begin{aligned}
\tilde{\mathcal{B}} &= \left(F'\left(\Sigma^{-1} \otimes X'X\right)F\right)^{-1}\left(F'\left(\Sigma^{-1} \otimes X'\right) vec(Z)\right),\\
\tilde{\Omega} &= \left(F'\left(\Sigma^{-1} \otimes X'X\right)F\right)^{-1},\\
\tilde{S} &= Z'Z - Z'X(X'X)^{-1}X'Z,\\
\tilde{R}_{dd} &= Z^{d\prime}X\left((X'X)^{-1} - I_{\mathcal{U}}\left(I'_{\mathcal{U}}X'XI_{\mathcal{U}}\right)^{-1}I'_{\mathcal{U}}\right)X'Z^{d},\\
Z &= Y - X^{\mathcal{R}}cM^{d},\\
\tilde{\nu} &= T - k.
\end{aligned}$$

Remark 1 (Jeffreys's prior without constraints) *Without constraints ($\mathcal{R} = \emptyset$), proposition 1 boils down to the usual multivariate normal–inverse Wishart distribution:*

$$\begin{aligned}
\mathcal{B} \mid \Sigma, Y, X &\sim \mathcal{N}\left(vec(\tilde{B}), \tilde{\Omega}\right),\\
\Sigma \mid Y, X &\sim \mathcal{W}^{-1}\left(\tilde{S}, \tilde{\nu}\right),
\end{aligned}$$

where:

$$\begin{aligned}
\tilde{B} &= \left(X'X\right)^{-1}\left(X'Y\right),\\
\tilde{\Omega} &= \Sigma \otimes \left(X'X\right)^{-1}.
\end{aligned}$$

3.2 Minnesota Prior

When the VAR is unrestricted, the conventional way to design a prior for the coefficient on variable i is: $B'_i \sim \mathcal{N}\left(\underline{B}'_i, \frac{1}{\kappa_i}\Sigma\right)$ where B_i is line i of B, with dimension $(1, n)$. We choose the restricted equivalent:

$$B'_i \sim \mathcal{N}\left(\underline{B}'_i, \frac{1}{\kappa_i}\Sigma\right), \quad i \in \mathcal{U}, \tag{17}$$

$$B^{o\prime}_i \sim \mathcal{N}\left(\underline{B}^{o\prime}_i, \frac{1}{\kappa_i}\left(\Sigma^{-1}\right)^{-1}_{oo}\right), \quad i \in \mathcal{R}. \tag{18}$$

We have the following prior on Σ:

$$\Sigma \sim \mathcal{W}^{-1}\left(\underline{S}, \underline{\nu}\right). \tag{19}$$

We implement this prior with dummy observations. For $i \in \mathcal{U}$, we can add the dummy observation $(\underline{Y}_i, \underline{X}_i) = (\sqrt{\kappa_i}\underline{B}_i, \sqrt{\kappa_i}e_i)$ where e_i is the row vector of dimension k whose element i is 1 and other elements are 0. The likelihood of that observation is proportional to: $|\Sigma|^{-1/2} \exp\left(-\frac{1}{2}\left(\underline{Y}_i - \underline{X}_i B\right)\Sigma^{-1}\left(\underline{Y}_i - \underline{X}_i B\right)'\right) \propto \mathcal{N}\left(\underline{B}_i, \frac{1}{\kappa_i}\Sigma; B_i\right)$. To implement the prior for $i \in \mathcal{R}$, we construct the following observation:

$$\underline{Y}_i = \sqrt{\kappa_i}\begin{pmatrix} c_i & \underline{B}_i^o \end{pmatrix} \quad \underline{X}_i = \sqrt{\kappa_i}e_i.$$

We notice that:

$$\underline{Y}_i - \underline{X}_i B = \sqrt{\kappa_i}\begin{pmatrix} c_i - c_i & \underline{B}_i^o - B_i^o \end{pmatrix} = -\sqrt{\kappa_i}\begin{pmatrix} 0 & B_i^o - \underline{B}_i^o \end{pmatrix}.$$

Then, the likelihood of that dummy observation is proportional to:

$$\begin{aligned} &|\Sigma|^{-1/2} \exp\left(-\frac{1}{2}\left(\underline{Y}_i - \underline{X}_i B\right)\Sigma^{-1}\left(\underline{Y}_i - \underline{X}_i B\right)'\right) \\ &\quad = |\Sigma_{dd}|^{-1/2}\left|(\Sigma^{-1})_{oo}^{-1}\right|^{-1/2} \exp\left(-\frac{1}{2}\left(B_i^o - \underline{B}_i^o\right)\left(\kappa_i\left(\Sigma^{-1}\right)_{oo}\right)\left(B_i^o - \underline{B}_i^o\right)'\right) \\ &\quad = |\Sigma_{dd}|^{-1/2} \times \mathcal{N}\left(\underline{B}_i^o, \frac{1}{\kappa_i}\left(\Sigma^{-1}\right)_{oo}^{-1}; B_i^o\right). \end{aligned}$$

So, the likelihood of the dummy observations for equations (17–18) is proportional to the density of the prior multiplied by $|\Sigma_{dd}|^{-k_{\mathcal{R}}/2}$ where $k_{\mathcal{R}}$ is the number of restricted coefficients in the debt equation:

$$p(\underline{Y}^B, \underline{X}^B \mid B, \Sigma) \propto |\Sigma_{dd}|^{-k_{\mathcal{R}}/2} \times p(B \mid \Sigma),$$

where $\underline{Y}^B$, $\underline{X}^B$ denote the stacked dummy observations for equations (17–18). There is one dummy observation per right-hand side variable, so the number of rows of $\underline{Y}^B$ and $\underline{X}^B$ is k. Then, we have the posterior for B and Σ:

$$\begin{aligned} p(B, \Sigma \mid Y, X) &\propto p(Y, X \mid B, \Sigma) \times p(B, \Sigma) \\ &\propto |\Sigma_{dd}|^{k_{\mathcal{R}}/2} \times p(Y, X, \underline{Y}^B, \underline{X}^B \mid B, \Sigma) \times p(\Sigma). \end{aligned} \tag{20}$$

We also implement equation (19) with dummy observations. Formally, we add $\underline{\underline{\nu}}$ times the following dummy observations: $\left(\underline{s}_j v_j, (0)_k\right)$, $1 \leq j \leq n$, where $\underline{s}_j$ is a scalar, v_j is a row vector of dimension n whose j^{th} element is 1 and other elements are 0 and $(0)_k$ is a row vector of dimension k whose elements are 0. The joint likelihood of those observations is proportional to the density of the prior distribution in equation (19):

$$\left(\prod_{j=1}^{n} |\Sigma|^{-1/2} \exp\left(-\frac{1}{2}\underline{s}_j^2 v_j \Sigma^{-1} v_j'\right)\right)^{\underline{\underline{\nu}}} = |\Sigma|^{-n\underline{\underline{\nu}}/2} \exp\left(-\frac{1}{2}\mathrm{tr}\left(\Sigma^{-1}\underline{\underline{\nu}}\sum_{j=1}^{n}\underline{s}_j^2(v_j'v_j)\right)\right)$$
$$\propto \mathcal{W}^{-1}\left(\underline{S}, \underline{\nu}; \Sigma\right),$$

where:

$$\underline{S} = \underline{\underline{\nu}} \begin{pmatrix} \underline{s}_1^2 & 0 & \vdots & 0 \\ 0 & \underline{s}_2^2 & \vdots & 0 \\ \dots & \dots & \vdots & \dots \\ 0 & 0 & \vdots & \underline{s}_n^2 \end{pmatrix}, \quad \underline{\nu} = n\underline{\underline{\nu}} - (n+1).$$

So, we can rewrite the posterior distribution in equation (20) into:

$$p(B, \Sigma \mid Y, X) \propto |\Sigma_{dd}|^{k_{\mathcal{R}}/2} \times p(\bar{Y}, \bar{X} \mid B, \Sigma), \tag{21}$$

where $\bar{Y}$ and $\bar{X}$ are the stacked data and dummy observations that implement equations (17–19). The number of rows of $\bar{Y}$ and $\bar{X}$ is $T + k + n\underline{\underline{\nu}}$.

Using steps similar to those of section 3.1, we can use equation (21) to obtain the posterior for B and Σ. Equation (15) gives us $p(\bar{Y}, \bar{X} \mid B, \Sigma)$. The number of degrees of freedom of Σ_{dd} and $(\Sigma^{-1})_{oo}^{-1}$ require special attention. Including dummies, the number of observations is now $T + k + n\underline{\underline{\nu}}$, so the exponents in equation (15) become $-(T + n\underline{\underline{\nu}} + k_{\mathcal{R}})/2 = -(T + \underline{\nu} + k_{\mathcal{R}} + n + 1)/2$ for $|\Sigma_{dd}|$ and $-(T + n\underline{\underline{\nu}} - 1)/2 = -(T + \underline{\nu} + n)/2$ for $\left|(\Sigma^{-1})_{oo}^{-1}\right|$. The presence of $|\Sigma_{dd}|^{k_{\mathcal{R}}/2}$ in equation (21) changes the exponent for $|\Sigma_{dd}|$ to: $-(T + \underline{\nu} + n + 1)/2$. Finally, the change of variables adds a factor of $|\Sigma_{dd}|^{n-1}$ to the density, so it changes the exponent to: $-(T + \underline{\nu} + n + 1)/2 + (n - 1) = -(T + \underline{\nu} - (n - 1) + 2)/2$. The sizes of Σ_{dd} and $(\Sigma^{-1})_{oo}^{-1}$ are 1 and $n - 1$ respectively, so we need to subtract $(1 + 1)/2$ and $(n - 1 + 1)/2$ from the exponents (and multiply by -2) to obtain the number of degrees of freedom: $T + \underline{\nu} - (n - 1)$ for Σ_{dd} and $T + \underline{\nu}$ for $(\Sigma^{-1})_{oo}^{-1}$. We arrive at the following proposition.

Proposition 2 (Minnesota prior) *With the Minnesota prior (17–19), the posterior distributions of B and Σ are:*

$$\mathcal{B} \mid \Sigma, \bar{Y}, \bar{X} \sim \mathcal{N}\left(\bar{\mathcal{B}}, \bar{\Omega}\right),$$
$$\Sigma_{dd}^{-1}\Sigma_{do} \mid \left(\Sigma^{-1}\right)_{oo}^{-1}, \bar{Y}, \bar{X} \sim \mathcal{MN}\left(\bar{S}_{dd}^{-1}\bar{S}_{do}, \bar{S}_{dd}^{-1}, \left(\Sigma^{-1}\right)_{oo}^{-1}\right),$$

$$\left(\Sigma^{-1}\right)_{oo}^{-1} \mid \bar{Y}, \bar{X} \sim \mathcal{W}^{-1}\left(\left(\bar{S}^{-1}\right)_{oo}^{-1}, \bar{\nu}\right),$$
$$\Sigma_{dd} \mid \bar{Y}, \bar{X} \sim \mathcal{W}^{-1}\left(\bar{S}_{dd} + \bar{R}_{dd}, \bar{\nu} - (n-1)\right),$$

where:

$$\bar{\mathcal{B}} = \left(F'\left(\Sigma^{-1} \otimes \bar{X}'\bar{X}\right) F\right)^{-1} \left(F'\left(\Sigma^{-1} \otimes \bar{X}'\right) vec(\bar{Z})\right),$$
$$\bar{\Omega} = \left(F'\left(\Sigma^{-1} \otimes \bar{X}'\bar{X}\right) F\right)^{-1},$$
$$\bar{S} = \bar{Z}'\bar{Z} - \bar{Z}'\bar{X}(\bar{X}'\bar{X})^{-1}\bar{X}'\bar{Z},$$
$$\bar{R}_{dd} = \bar{Z}^{d\prime}\bar{X}\left((\bar{X}'\bar{X})^{-1} - I_{\mathcal{U}}\left(I_{\mathcal{U}}'\bar{X}'\bar{X}I_{\mathcal{U}}\right)^{-1} I_{\mathcal{U}}'\right)\bar{X}'\bar{Z}^d,$$
$$\bar{Z} = \bar{Y} - \bar{X}^{\mathcal{R}} c M^d,$$
$$\bar{\nu} = T + \underline{\nu}.$$

Remark 2 (Minnesota prior without constraint) *Remark 1 applies to the Minnesota prior with an appropriate change of notation.*

4 Posterior Mode

We want to find the mode of the posterior distribution given the observed data. Since some data may be missing, we need to integrate over the missing data:

$$\begin{aligned} p(B, \Sigma \mid Y^o, X^o) &= \int_{Y^m, X^m} p(B, \Sigma \mid Y, X) p(Y^m X^m \mid Y^o, X^o) dY^m dX^m \\ &= E^m\left[p(B, \Sigma \mid Y, X)\right]. \end{aligned}$$

4.1 Mode with Jeffreys's Prior

$p(B, \Sigma \mid Y, X)$ can be deduced from proposition 1:

$$\begin{aligned} \tilde{\mathcal{P}}^0 =& p(B, \Sigma \mid Y, X) \\ \propto& \left|\tilde{\Omega}\right|^{-1/2} \exp\left(-\frac{1}{2}\mathrm{tr}\left(\left(\mathcal{B} - \tilde{\mathcal{B}}\right)' \tilde{\Omega}^{-1}\left(\mathcal{B} - \tilde{\mathcal{B}}\right)\right)\right) \\ &\times \left|\tilde{S}_{dd}^{-1}\right|^{-(n-1)/2} \left|\left(\Sigma^{-1}\right)_{oo}^{-1}\right|^{-1/2} \\ &\times \exp\left(-\frac{1}{2}\mathrm{tr}\left(\left(\Sigma^{-1}\right)_{oo}\left(\Sigma_{dd}^{-1}\Sigma_{do} - \tilde{S}_{dd}^{-1}\tilde{S}_{do}\right)' \tilde{S}_{dd}\left(\Sigma_{dd}^{-1}\Sigma_{do} - \tilde{S}_{dd}^{-1}\tilde{S}_{do}\right)\right)\right) \\ &\times \left|\tilde{S}_{dd} + \tilde{R}_{dd}\right|^{(\tilde{\nu}+k_{\mathcal{R}}-(n-1))/2} \left|\Sigma_{dd}\right|^{-(\tilde{\nu}+k_{\mathcal{R}}+n+1)/2} \end{aligned}$$

$$\times \exp\left(-\frac{1}{2}\text{tr}\left(\Sigma_{dd}^{-1}\left(\tilde{S}_{dd}+\tilde{R}_{dd}\right)\right)\right)$$
$$\times\left|\left(\tilde{S}^{-1}\right)_{oo}^{-1}\right|^{\tilde{\nu}/2}\left|\left(\Sigma^{-1}\right)_{oo}^{-1}\right|^{-(\tilde{\nu}+(n-1)+1)/2}$$
$$\times \exp\left(-\frac{1}{2}\text{tr}\left(\left(\Sigma^{-1}\right)_{oo}\left(\tilde{S}^{-1}\right)_{oo}^{-1}\right)\right). \tag{22}$$

We note that we have reverted the change of variable by multiplying the density by $|\Sigma_{dd}|^{-(n-1)}$.

We know $|\tilde{\Omega}^{-1}|$ from equation (9) and we can use it to simplify the determinants of $\tilde{\mathcal{P}}^0$ to:

$$\left|\tilde{S}_{dd}\right|^{(n-1)/2}\left|\tilde{S}_{dd}+\tilde{R}_{dd}\right|^{(\tilde{\nu}+k_{\mathcal{R}}-(n-1))/2}\left|\left(\tilde{S}^{-1}\right)_{oo}^{-1}\right|^{\tilde{\nu}/2} \times\left|X'X\right|^{(n-1)/2}\left|I_{\mathcal{U}}'X'XI_{\mathcal{U}}\right|^{1/2}\left|\Sigma\right|^{-(\tilde{\nu}+k+n+1)/2}. \tag{23}$$

To find the first-order condition with respect to $\mathcal{B}$, it will be convenient to notice that the only term of $\tilde{\mathcal{P}}^0$ that depends on $\mathcal{B}$ is:

$$\exp\left(-\frac{1}{2}\text{tr}\left(\left(\mathcal{B}-\tilde{\mathcal{B}}\right)'\tilde{\Omega}^{-1}\left(\mathcal{B}-\tilde{\mathcal{B}}\right)\right)\right). \tag{24}$$

To find the first-order condition with respect to Σ, it will be convenient to use the original formulation of the trace term in $\tilde{\mathcal{P}}^0$:

$$\text{tr}\left(\Sigma^{-1}\tilde{\Psi}\right),\qquad \tilde{\Psi}=(Y-BX)'(Y-BX). \tag{25}$$

The first-order conditions with respect to $\mathcal{B}$ and Σ yield proposition 3.

Proposition 3 (mode with Jeffreys's prior) *With Jeffreys's prior (16), the mode of the posterior distribution is characterized by:*

$$\mathcal{B}=\left(E^m\left[\tilde{\mathcal{P}}F'\left(\Sigma^{-1}\otimes(X'X)\right)F\right]\right)^{-1}\left(E^m\left[\tilde{\mathcal{P}}F'\left(\Sigma^{-1}\otimes X'\right)vec\left(Y-X^{\mathcal{R}}cM^d\right)\right]\right),$$
$$\Sigma=\frac{1}{\tilde{\nu}+k+n+1}\frac{E^m\left[\tilde{\mathcal{P}}\tilde{\Psi}\right]}{E^m\tilde{\mathcal{P}}},$$

where:

$$\tilde{\Psi}=(Y-BX)'(Y-BX),$$
$$\tilde{\mathcal{P}}=\left|\tilde{S}_{dd}\right|^{(n-1)/2}\left|\tilde{S}_{dd}+\tilde{R}_{dd}\right|^{(\tilde{\nu}+k_{\mathcal{R}}-(n-1))/2}\left|\left(\tilde{S}^{-1}\right)_{oo}^{-1}\right|^{\tilde{\nu}/2}\left|X'X\right|^{(n-1)/2}\left|I_{\mathcal{U}}'X'XI_{\mathcal{U}}\right|^{1/2}$$

$$\times \exp\left(-\frac{1}{2} tr\left(\Sigma^{-1}\tilde{\Psi}\right)\right).$$

Remark 3 (mode without missing observations) *In the particular case where there are no missing observations, the mode of the posterior distribution is simply given by:*

$$vec(B) = \left(F'\left(\Sigma^{-1}\otimes(X'X)\right)F\right)^{-1}\left(F'\left(\Sigma^{-1}\otimes X'\right) vec\left(Y - X^{\mathcal{R}}cM^{d}\right)\right),$$
$$\Sigma = \frac{\tilde{\Psi}}{\tilde{\nu}+k+n+1}.$$

4.2 Mode with Minnesota Prior

As in section 3.2, the argument is similar to the case of Jeffreys's prior, with an adjustment to the degrees of freedom of Σ_{dd}. As a result, equation (23) becomes:

$$\mid \bar{S}_{dd}\mid^{(n-1)/2}\mid \bar{S}_{dd}+\bar{R}_{dd}\mid^{(\bar{\nu}-(n-1))/2}\mid (\bar{S}^{-1})^{-1}_{oo}\mid^{\bar{\nu}/2}$$
$$\times \mid \bar{X}'\bar{X}\mid^{(n-1)/2}\mid I'_{\mathcal{U}}\bar{X}'\bar{X}I_{\mathcal{U}}\mid^{1/2}\mid \Sigma_{dd}\mid^{k_{\mathcal{R}}/2}\mid \Sigma\mid^{-(\bar{\nu}+k+n+1)/2}. \quad (26)$$

and $\tilde{\Psi}$ is replaced by: $\bar{\Psi} = \left(\bar{Y}-B\bar{X}\right)'\left(\bar{Y}-B\bar{X}\right)$.

The first-order conditions with respect to $\mathcal{B}$ and Σ yield proposition 4.

Proposition 4 (mode with Minnesota prior) *With the Minnesota prior (17–19), the mode of the posterior distribution is characterized by:*

$$\mathcal{B} = \left(E^m\left[\bar{\mathcal{P}}F'\left(\Sigma^{-1}\otimes(\bar{X}'\bar{X})\right)F\right]\right)^{-1}\left(E^m\left[\bar{\mathcal{P}}F'\left(\Sigma^{-1}\otimes\bar{X}'\right)vec(\bar{Z})\right]\right),$$
$$\Sigma_{dd} = \frac{1}{\bar{\nu}+k-k_{\mathcal{R}}+n+1}\frac{E^m\left[\bar{\mathcal{P}}\bar{\Psi}_{dd}\right]}{E^m\bar{\mathcal{P}}},$$
$$\Sigma_{do} = \frac{1}{\bar{\nu}+k-k_{\mathcal{R}}+n+1}\frac{E^m\left[\bar{\mathcal{P}}\bar{\Psi}_{do}\right]}{E^m\bar{\mathcal{P}}},$$
$$\Sigma_{oo} = \frac{1}{\bar{\nu}+k+n+1}\left(\frac{E^m\left[\bar{\mathcal{P}}\bar{\Psi}_{oo}\right]}{E^m\bar{\mathcal{P}}}+k_{\mathcal{R}}\Sigma_{od}\Sigma_{dd}^{-1}\Sigma_{do}\right),$$

where:

$$\bar{Z} = \bar{Y} - \bar{X}^{\mathcal{R}}cM^{d},$$
$$\bar{\Psi} = \left(\bar{Y}-B\bar{X}\right)'\left(\bar{Y}-B\bar{X}\right),$$
$$\bar{\mathcal{P}} = \left|\bar{S}_{dd}\right|^{(n-1)/2}\left|\bar{S}_{dd}+\bar{R}_{dd}\right|^{(\bar{\nu}-(n-1))/2}\left|\left(\bar{S}^{-1}\right)^{-1}_{oo}\right|^{\bar{\nu}/2}\left|\bar{X}'\bar{X}\right|^{(n-1)/2}\left|I'_{\mathcal{U}}\bar{X}'\bar{X}I_{\mathcal{U}}\right|^{1/2}$$
$$\times \exp\left(-\frac{1}{2}tr\left(\Sigma^{-1}\bar{\Psi}\right)\right).$$

Remark 4 (mode without missing observations) *Remark 3 applies to the Minnesota prior with appropriate changes of formula.*

Remark 5 (computational difficulty) *When many data points are missing (i.e., when Y^m and X^m are of high dimension), we found that using the draws of Y^m and X^m obtained from the state smoother was unreliable, even if the BVAR is unconstrained. So we only report the impulse response function (IRF) at the mode when no data is missing and we report the pointwise median IRF otherwise.*

5 Related Approaches

The posterior distribution could also be estimated by Gibbs sampling: (i) sample B conditional on Σ and (ii) sample Σ conditional on B. For example, Jarociński and Karadi (2020) estimate a BVAR with all coefficients restricted to 0 in the first equation. This approach would be perfectly valid but computationally slower, since the draws are no longer independent and more draws must be taken. In our application, some data is missing, so we must run a Kalman filter and state smoother at every draw. The additional computational time of every draw can make an increased number prohibitive.

Antolín-Díaz et al. (2025) propose a constrained BVAR with restrictions on the autoregressive coefficients and the covariance matrix. Their procedure requires a custom importance sampling algorithm. We do not have restrictions on the covariance matrix, which allows us to derive analytical expressions for its posterior distribution and avoids the need for importance sampling.

In short, our approach maximizes analytical tractability and minimizes computational complexity, at the cost of lengthy derivations.

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